

MODELLING OF TURBULENT BUBBLE FLOW IN THE BOUNDARY LAYER

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The work has been conducted in order to determine the first principles, which govern the gas phase distribution in vertical bubbly flow in the boundary layer on a plate. Correct modelling of the gas phase distribution is of primary importance for pressure drop predictions and heat and mass transfer calculations. Bearing in mind, that in such a case in the flow there are multidimensional effects, which occur at specific flow regimes of bubbly flow it is so difficult to capture the physics of such behaviour. In the case of upward bubbly flow there are various distributions of void fraction observed taking a form which peaks in the core of the flow, so called core peaking, or has a peak close to the wall and is named wall peaking. Several multidimensional mechanisms have been proposed to explain these mechanisms, however so far none of them is capable of fully describe this behaviour.

In this paper an original proposal to modelling the void fraction distributions in the bubbly flow in the boundary layer is given. The proposed model is based on a distinction of two different rotations acting on a bubble, one due to sort of Magnus force and the second one due to the wake flow of travelling bubbles. Depending on the magnitude of the rotations, the resultant force is moving the bubble either towards the wall or the centre. Additionally, the influence of buoyancy forces is considered on the shear stress distribution across the boundary layer.

MODEL OF A TURBULENT TWO PHASE FLOW

In [1,2] presented has been an analysis of two-phase bubbly flow over a flat plate based on energetical considerations of the dissipation process in two-phase bubble flow. The model is based on the following assumptions: turbulence exists only in the liquid phase, dispersed phase (bubbles) occupy some volume of the flow and influence the momentum and turbulence of liquid phase, there is no motion of the gaseous phase inside the bubbles, fluid motion is fully developed, co-linear bubble velocity and force acting on the bubble and the surface void fraction is equal to the volumetric one.

The problem is considered as one-dimensional, where some control volume V is containing a number of vapour bubbles n . Defining a specific energy dissipation of the flow as a ratio of power dissipated by the bubbles in the control volume, where the dissipation power can be expressed as a product of the total force acting on the bubbles and relative bubble velocity. The force acting on the bubble is a friction force, which in the present work will be expressed as an aerodynamical force. Using the above assumptions, equation expressing the energy dissipation due to the presence of bubbles takes a form

$$e_b = \frac{N}{V} = \frac{nF_b u_R}{V} = \frac{6\phi F_b u_R}{\pi D_b^3} = \frac{3}{4} \frac{\phi \rho_L C_D u_R^3}{D_b} \quad (1)$$

Let's assume an idea of an equivalent two-phase flow, which is deemed as a flow with single phase properties corresponding to the properties of two-phase flow. Let's define the dissipation energy of two-phase flow as a power lost due to friction by an arbitrary layer isolated from the

velocity profile with respect to the control volume. Dissipation of the flow in the control volume is expressed by the ratio of a square of the shear stress in the continuous phase to the turbulent viscosity

$$e_{TP} = \frac{N_{TP}}{V} = \frac{\tau_{TP} S w_{TP}}{S \delta} = \frac{\tau_{TP} w_{TP}}{\delta} = \frac{\tau_{TP}^2}{\mu_t} = \frac{(1-\phi)^2 \tau_l^2}{\mu_t} \quad (2)$$

where τ_{TP} describe the shear stress in two-phase flow. Dissipation of the two-phase flow in the control volume of arbitrary layer isolated from the flow can be written analogically to (2) as:

$$e_e = \frac{\tau_e^2}{\mu_t} \quad (3)$$

Turbulent viscosity is modelled using the simplest turbulence model, i.e. Prandtl mixing length model. It is postulated that the total dissipation energy in the flow is a sum of dissipation from shearing flow and dissipation from the bubbles in the form, the foundation of the present model

$$e_e = e_{TP} + e_b \quad (4)$$

In our analysis we will also use the following quantities serving in dimensionless analysis

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} \quad w^+ = \frac{w}{u_\tau} \quad y^+ = \frac{y u_\tau}{\nu} \quad \tau^+ = \frac{\tau_e}{\tau_w} \quad (5)$$

Substituting (1), (2) and (3) to (4) we obtain a relation linking the shear stress in two-phase flow with the turbulent stresses of continuous phase and energy of dissipation from the presence of bubbles, i.e. the model of the two-phase flow. Substituting for the turbulent viscosity and casting into a non-dimensional form we get

$$\frac{\tau^+{}^2}{\kappa^4} = (1-\phi)^2 y^{+4} \left(\frac{dw^+}{dy^+} \right)^4 + \frac{3}{4} \frac{\phi C_D u_R^3}{D_b \kappa^2} \frac{\nu}{u_\tau^4} y^{+2} \left(\frac{dw^+}{dy^+} \right) \quad (6)$$

This expression obeys the asymptotic condition, i.e. when the void fraction, $\phi=0$, we have a single phase flow of liquid alone. The above result can be cast into a more general form which describes a very important issue, i.e. that the stress in the two-phase flow is a sum of the squares of stresses coming from shear stress in liquid with the presence of bubbles and interaction of bubbles on the liquid, a geometrical summation rather than algebraic one

$$\tau_e^2 = \tau_{TP}^2 + \tau_b^2 \quad (7)$$

MECHANISM OF LATERAL MOTION OF BUBBLES

Mechanism of lateral migration of bubbles in the vicinity of the wall has yet to be investigated properly. It results from the experimental data that the distribution of dispersed phase in the channel cross-section stems from the balance of lateral and drag forces [3], i.e. the lift force F_b is balanced by the lateral drag force F_D and the inertia force. The force balance on the bubble takes form:

$$m_b \frac{d g_b}{dt} = F_b - F_D \quad (8)$$

where m_b is a bubble virtual mass, which is modelled as a half of the mass of displaced liquid. It has been calculated as a thin layer of liquid surrounding the bubble of thickness δ . Sought added mass has been calculated and is equal to $\rho_L \pi d_b^3 / 12$. Additionally in (8) $\mathfrak{G}_b$ is a lateral bubble velocity. The lift force can be expressed as

$$F_b = C_L \frac{\pi}{6} \rho_L d_b^3 u_R \omega \quad (9)$$

The relative bubble velocity, u_R , is approximated by the bubble terminal rise velocity in a quiescent container, $u_R = u_b - u_L \approx u_\infty$. Such relation is applicable in the case of small void fractions.

The angular velocity of considered bubble resulting from the velocity field is $\omega = \frac{2\Delta u_L}{d_b} \approx 2 \frac{\partial u_L}{\partial r}$.

There is however another influence on the bubble rotation, which has yet to be considered in the literature. It is caused by the fact that the bubble in vertical upward motion releases some space behind it, which is subsequently filled by the liquid from the bubble front [2]. If we consider n bubbles in a given volume, then each bubble occupies an imaginary channel with the area of $1/n$. Channels have different surfaces due to the fact that there is a lateral distribution of dispersed phase in the channel. A continuity equation can be written for a selected plane perpendicular to the flow. On such basis a velocity, opposite in direction to the liquid velocity, can be derived, which is equal $\mathfrak{G} = dy/dt$ [2]. This velocity gives rise to another angular velocity, which subsequently depending on the concentration profile, i.e. *wall peaking* or *core peaking*, acts in the same or opposite direction as rotation stemming from velocity profile. In the case of two rotations with opposite signs we can observe the motion in one or another way. Such motion is identified by the bubble diameter, i.e. for a certain range of bubble diameters the motion is towards the wall and in others towards the core of the flow, which has experimental confirmation. The presence of a second rotation should contribute to explanation of such behaviours. In connection to this fact the resultant circulation in (9) ought to be changed, as now it consists of two components, i.e. $\omega = \omega_1 + \omega_2$. Circulation can be cast as follows

$$\omega = \omega_1^0 + \omega_2^0 = \left\{ \frac{2 u_\tau}{\kappa y} - \frac{2 u_\infty}{(1-\alpha)^2} \left(\frac{d\alpha}{dy} \right) \right\} \quad (10)$$

MODEL OF VOID FRACTION

Lateral drag force can be written in the form:

$$F_D = C_D A \frac{\rho_L \mathfrak{G}_b^2}{2} = C_D \frac{\pi d_b^2 \rho_L \mathfrak{G}_b^2}{8} \quad (11)$$

In order to determine this force we need to assume friction factor C_D . For the bubble Reynolds number from the range $0.5 \div 1000$ a relation $C_D = 24 / \text{Re}_b (1 + 0.15 \text{Re}_b^{0.687})$ is normally used. From equation (8) we can determine a differential equation describing the distribution of lateral velocity

$$\frac{d \mathfrak{G}_b}{dy} = \frac{36 \nu_L}{d_b^2} (1 + 0.15 \text{Re}_b^{0.687}) - \frac{4 C_L u_R}{\mathfrak{G}_b} \left[\frac{u_\tau}{\kappa y} - \frac{u_\tau}{(1-\alpha)^2} \frac{d\alpha}{dy} \right] \exp \left(- \frac{120 \nu_L}{d_b^2 \mathfrak{G}_b} y \right) \quad (12)$$

Expression (12) contains two unknowns, namely $\mathfrak{G}_b$ and the void fraction α . Mass transfer equation will supplement the model and enable determination of $\mathfrak{G}_b$ and α .

$$\alpha \mathcal{G}_b = -D_b \frac{d\alpha}{dy} \quad (13)$$

where D_b is a local bubble diffusivity.

Flow in the boundary layer

In [1,2] it was assumed, that in the boundary layer the shear stress is constant $\tau=\tau_w=\text{const}$ which is a natural assumption in the boundary layer, where the buoyancy forces are neglected. However, if we want to consider the direction of vertical motion then we have to consider the influence of buoyancy forces on the velocity profile. Performing a force balance on an imaginary element of liquid containing bubbles then we obtain the following force balance between the weight of the control volume $G=\rho g dx dy$ and the buoyancy force $B=\rho_L g dx dy$:

$$-d\tau_e dx + g \rho_L dy dx - \rho g dy dx = 0 \quad (14)$$

where $\rho_e = \rho_L(1-\varphi) + \rho_G\varphi$. From (14) we derive the distribution of stress across the boundary

$$\frac{d\tau_e}{dy} = -g \left(1 - \frac{\rho_G}{\rho_L}\right) \rho_L \varphi \quad (15)$$

As we see the solution is based on solving of three differential equations. Such an approach has not been found by the author in the literature. The results have been obtained using MATHCAD5+ with its standard procedure Runge-Kutta procedure with adaptive stepsize.

COMPARISON AGAINST EXPERIMENTAL DATA

Calculations performed using the postulated model have been compared against the experimental data of Marié et al for the case of upward bubbly flow on a flat plate. Experimental data was available for void fraction distribution and velocity profiles. Results are presented in Figures 1 and 2.

As can be seen from the presented graphs, a new simple theory leads to surprisingly good results, when velocity profiles are compared, see Fig. 1. There is, however, a quantitative discrepancy between the experiment and the results, which is strongly influenced by the choice of the boundary condition on the velocity profile. In Fig. 1 presented is also a plot of velocity profile when the shear stress across the boundary layer was assumed constant and $\tau^+=1$. This distribution is shifted down compared to the distribution with variable shear stress. We should also note that this solution does not deflect the velocity profile in the core region. In the figure presented is also a plot of a universal velocity profile for single phase flow. It is shifted up compared to the experimental data and a model. Presented results show that the mean velocity distribution exhibits three distinct zones: a viscous sublayer, a logarithmic zone and a wake. The present model gives a good consistency with the experiment in the logarithmic zone and a wake.

The next issue studied here was the void fraction distribution, which is presented in Fig. 2. As can be seen there is a remarkable good qualitative agreement between the model and experimental data. Proposed model is capable of capturing the wall peaking in upward flow. However, the quantitative agreement is less satisfactory, as the predicted peak is shifted towards core of the flow, but it must be stressed that the model is very sensitive to the bubble diameter.