# AN ECCENTRIC CELL APPROACH TO THE CLOSURE FOR THE TWO FLUID MODEL


G. Espinosa[+], A. Soria and E. Salinas-Rodríguez

Departamento de Ingeniería y Procesos e Hidráulica, Universidad Autónoma Metropolitana-Iztapalapa, Av. Michoacán y La Purísima s/n, Col. Vicentina, C.P. 09340, México D.F.
+Tel: (52) 5804 46 45; Fax: (52) 5804 49 00; E-mail: gepe@xanum.uam.mx



A one-dimensional, time-dependent, isothermal, incompressible, Newtonian fluid, two-phase volume-averaging model was developed to study momentum interaction effects in vertical ducts with bubble flow regime. For the evaluation of the averaged description, potential inviscid flow around bubbles was considered in order to get closure relationships, using an eccentric cell model approach.


## INTRODUCTION

Consider the two-fluid balance equations of two-phase flow. The system is composed of a continuous liquid phase (water) and a disperse gas phase (air) made up of a swarm of spherical bubbles. The system is taken as isothermal and the vertical duct with a diameter much greater than the size of the individual bubbles. Both phases are considered locally incompressible and without interfacial mass transfer. Interfacial momentum transfer is allowed and a constant surface tension is given. The mixture of gas and liquid is assumed to be far away from the solid walls of the duct, in such a way that wall effects can be neglected.

The three-dimensional averaged transport equations for adiabatic air-water, are given by Espinosa and Soria[1]:

Averaged mass balance

$$\overbrace{\frac{\partial \varepsilon_k}{\partial t}}^{accumulation} + \overbrace{\varepsilon_k \nabla\cdot\langle \mathbf{v}_k\rangle^k + \langle \mathbf{v}_k\rangle^k\cdot\nabla\varepsilon_k}^{convection} = 0 \tag{1}$$

Average momentum balance

$$\begin{aligned} &\overbrace{\rho_k \frac{\partial}{\partial t}\left(\varepsilon_k\langle \mathbf{v}_k\rangle^k\right)}^{accumulation} + \overbrace{\rho_k\nabla\cdot\left(\varepsilon_k\langle \mathbf{v}_k\rangle^k\langle \mathbf{v}_k\rangle^k\right)}^{convection} + \overbrace{\varepsilon_k\nabla\langle p_k\rangle^k}^{pressure} + \overbrace{\varepsilon_k\rho_k\mathbf{g}}^{gravity} \\ &= \underbrace{\langle\Delta p_{km}\rangle\nabla\varepsilon_k}_{\text{difference of the averages of the pressure}} \; - \underbrace{\rho_k\nabla\cdot\left(\varepsilon_k\langle\tilde{\mathbf{v}}_k\tilde{\mathbf{v}}_k\rangle^k\right)}_{dispersion} + \underbrace{\mathbf{M}_{km}}_{\text{interfacial force}} \end{aligned} \tag{2}$$

where $k$ and $m$ ($k \neq m$) denotes either the gas ($k = g$) or the liquid ($k = l$) phase; $\langle\ \rangle^k$ represents an averaging operator; $\rho_k$, $\mathbf{v}_k$, $p_k$, are the local variables in the $k$-phase representing density, velocity vector and pressure, respectively, $g$ is the acceleration of gravity vector and $\varepsilon_k$ is the void fraction in the $k$-phase.

The difference between the interfacial average pressure and the intrinsic average pressure is given by[2]:

$$\langle \Delta p_{km} \rangle = \langle p_k \rangle_{km} - \langle p_k \rangle^k \tag{3}$$

The interfacial force per unit volume applied on phase $k$ is defined by:

$$\mathbf{M}_{km} \equiv -\frac{1}{V}\int_{A_{km}} \mathbf{n}_{km}\,\tilde{p}_{km}\,dA + \frac{1}{V}\int_{A_{km}} \mathbf{n}_{km}\cdot\mathbf{T}_{km}\,dA \tag{4}$$

where $n_{km}$ is the unit normal vector at the interface pointing out of $k$-phase, the spatial deviations of the pressure is given by

$$\tilde{p}_k = p_k - \langle p_k \rangle^k - \langle \Delta p_{km} \rangle \tag{5}$$

and the viscous stress tensor for incompressible flow is given by

$$\mathbf{T}_{km} = \mu_k \left(\nabla \mathbf{v}_k + \nabla \mathbf{v}_k^T\right)\Big|_{km} \tag{6}$$

In order to solve the equations (1) and (2), it is imperative to develop appropriate closure relationships for the terms $\langle \Delta p_{km} \rangle$, $\langle \tilde{\mathbf{v}}_k \tilde{\mathbf{v}}_k \rangle^k$ and $\mathbf{M}_{km}$. This is the purpose of the present work considering an eccentric cell model.

## MODEL DEVELOPED

According with Espinosa[3] the first integral of the equation (4) can be rewritten as:

$$\begin{aligned}\frac{1}{V}\int_{A_{lg}} \tilde{p}_{lg}\mathbf{n}_{lg}\,dA = &-\rho_l \nabla\cdot\left(\varepsilon_l \langle \tilde{\mathbf{v}}_l \tilde{\mathbf{v}}_l \rangle^l\right) + \frac{1}{2}\rho_l \varepsilon_l \nabla\left[\varepsilon_l^{-1}\nabla\cdot\left(\varepsilon_l \langle \tilde{\phi}\,\tilde{\mathbf{v}}_l \rangle^l\right)\right] \\ &- \rho_l \varepsilon_l \frac{\partial \mathbf{m}}{\partial t} - \frac{1}{2}\rho_l \varepsilon_l \nabla\left[\left(\langle \mathbf{v}_l \rangle^l + \langle \mathbf{v}_g \rangle^g\right)\cdot\mathbf{m}\right] + \langle \Delta p_{lg} \rangle \nabla \varepsilon_l\end{aligned} \tag{7}$$

where

$$\mathbf{m} = \frac{1}{\varepsilon_l V}\int_{A_{lg}} \tilde{\phi}\,\mathbf{n}_{lg}\,dA \tag{8}$$

where $\tilde{\phi}$ represents the spatial deviations of the potential velocity given by Gray[4]

$$\tilde{\phi} = \phi - \langle \phi \rangle^l \tag{9}$$

In order to obtain the averaging velocity potential the eccentric cell model is applied. The process under consideration is illustrated in Figure 1, where the phase sphere has a radius $a$ and the cell is a sphere of radius $b$, thus $(a/b)^3$ is equal to the volume fraction on the phase sphere. The sphere is located at a relative distance $\gamma$ with respect to the centroid. When a cell model is used to approximate a volume average, it is assumed that many spheres exist within the averaging volume. Cells are defined about each individual sphere. Hydrodynamic interactions were assumed to be negligible for sparse dispersions.

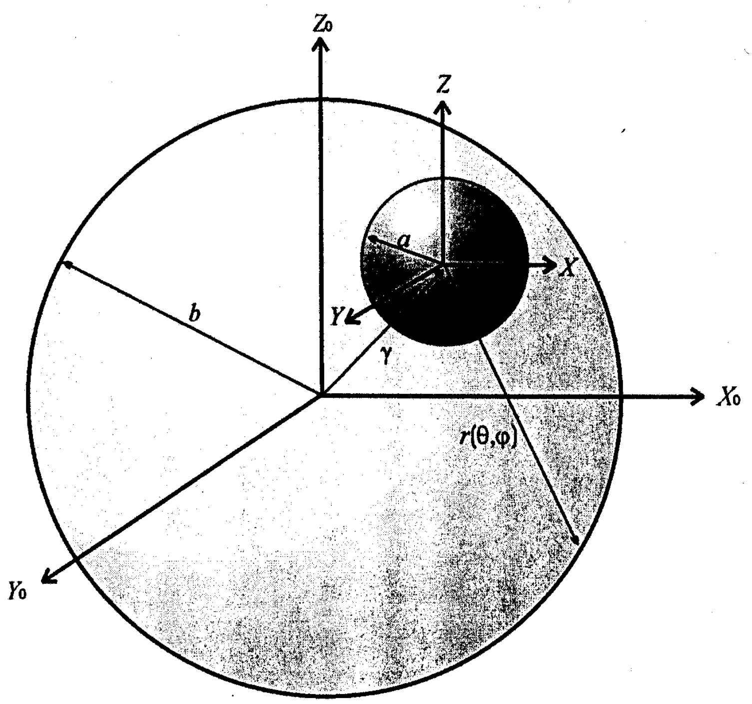


**Figure 1.** Eccentric cell model

Under the foregoing considerations the averaging velocity potential is given by

$$\langle \phi \rangle^l = \frac{1}{V_l} \int_0^{2\pi} \int_0^{\pi} \int_a^{r(\theta,\varphi)} \phi(r,\theta) r^2 \, dr \sin\theta \, d\theta \, d\varphi \tag{10}$$

where $V_l = 4/3\pi(b^3 - a^3)$ and $r(\theta, \varphi)$ is the eccentric radius, which depends on the bubble position within the cell (Figure 1).

Interfacial drag forces are obtained from the integral over the interfacial area for the viscous stress. This term is defined by Lahey[5] as:

$$\frac{1}{V} \int_{A_{lg}} \mathbf{n}_{lg} \cdot \mathbf{T}_{gl} \, dA = -\frac{1}{V} \int_{A_{lg}} \mathbf{n}_{gl} \cdot \mathbf{T}_{gl} \, dA \equiv \varepsilon_g \, \mathbf{F}_D \tag{11}$$

where $\mathbf{F}_D$ is the interfacial drag force. For a bubbly flow with uniform bubble size distribution, this force is given by[6]:

$$\mathbf{F}_D = \frac{3}{8}\rho_l \frac{C_D}{R_b}\left(\langle \mathbf{v}_g \rangle^g - \langle \mathbf{v}_l \rangle^l\right)\left|\langle \mathbf{v}_g \rangle^g - \langle \mathbf{v}_l \rangle^l\right| \tag{12}$$

where $R_b$ is the equivalent bubble radius, $C_D$ is the drag coefficient. Ishii and Mishima[6] report a detailed analysis on the two-phase drag coefficient.

## RESULTS AND DISCUSSION

The following results are obtained with the eccentric cell model. The first result is:

$$\mathbf{m} = \varepsilon_l^{-1} C_{IME}(\varepsilon_g)\mathbf{v}_r \tag{13}$$

where $\mathbf{v}_r$ is the relative velocity vector and $C_{IME}(\varepsilon_g)$ defined as added mass coefficient, which in this eccentric cell model is not constant anymore but a function of the eccentricity and the void fraction gradient. This result is important because includes an additional contribution to the usual term governed by the gradient of the void fraction. When the gradient is positive the coefficient is greater than $\varepsilon_g/2$, while for negative gradient the coefficient is less $\varepsilon_g/2$. If $r(\theta,\varphi) = b$ the result is reduced to $\varepsilon_g/2$, which corresponds to the concentric cell model. Pauchon and Smereka[7] also obtain this result.

The intrinsic average of the dyadic product of velocity deviations is given by

$$\langle \tilde{\mathbf{v}}_l \tilde{\mathbf{v}}_l \rangle^l = \mathbf{K}(\varepsilon_g)\varepsilon_g v_r^2 \tag{14}$$

where $v_r$ is the modulus of the relative velocity and **K** is second-order tensor given by

$$\mathbf{K}(\varepsilon_g) = \frac{\mathbf{K}_C}{1-\varepsilon_g} - \frac{a^6}{4V_l\varepsilon_g}\left(\frac{1}{3}\mathbf{B} + \frac{1}{V_l}\mathbf{d}_2\mathbf{d}_2\right) \tag{15}$$

where $\mathbf{K}_C$ is also a typical second-order tensor, defined by Wallis[8] for a concentric cell model, the vector $\mathbf{d}_2$ and the tensor $\mathbf{B}$ are functions of the eccentric radius.

In Figure 2 a comparison for the coefficient $K_{zz}$ for two eccentric conditions and for the Pauchon and Banerjee's work[9] is shown. The behaviour reported in[9] is strongly dependent on the void fraction, while the presently worked eccentric cell model is a smoother function of void fraction for $\gamma^*(=\gamma/b) = 0.0693$ at $K_{zz} \approx 0.3$. For $\gamma^* = 0.4503$ a maximum close to $\varepsilon_g \approx 0.343$ and a further decrease of $K_{zz}$ were observed.

## CONCLUSIONS

Using the eccentric cell model, first and second order effects for the void fraction were apparent, changing the structure of the non-linear differential equation.

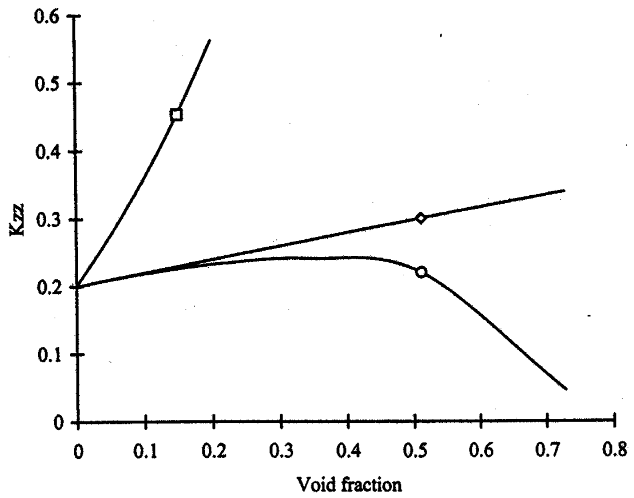


**Figure 2**. Comparison of the coefficients $K_{zz}$. (◊) $\gamma^* = 0.0693$ and (□) $\gamma^* = 0.4503$; (O) Pauchon and Banerjee[9].

## REFERENCES

1. Espinosa-Paredes, G. and Soria, A., Method of Finite Difference Solutions to the Transient Bubbly Air-Water Flows, *Int. J. Num. Method. Fluids*, Vol. 26, pp. 1155-1180, 1998.
2. Pauchon, C. and Banerjee, S., Interphase Momentum Interaction Effects in the Averaged Multifield Model. Part I: Void Propagation in Bubbly Flows, *Int. J. Multiphase Flow*, Vol. 12, pp. 559-573, 1986.
3. Espinosa-Paredes, G., Ondas Cinemáticas en Reactores BWR, Ph. D. Thesis, Universidad Autónoma Metropolitana-Iztapalapa, México, 1998.
4. Gray, W.G., A Derivation of the Equations for Multiphase Transport, *Chem. Engng. Sci.*, Vol. 30, pp. 229-233, 1975.
5. Lahey, R.T., Void Wave Propagation Phenomena in Two-Phase Flow, *A.I.Ch.E. J.*, Vol. 37, pp. 123-135, 1991.
6. Ishii, M. and Mishima, K., 1984, Two-fluid Model and Hydrodynamic Constitutive Relations, *Nuclear Engineering and Design*, **82**, 107-126.
7. Pauchon, C. and Smereka, P., 1992, Momentum Interactions in Dispersed Flow: An Averaging and A Variational Approach, *Int. J. Multiphase Flow,* **18**, 65-87
8. Wallis, G.B., 1989, Inertial Coupling in Two-Phase Flow: Macroscopic Properties of Suspensions in An Inviscid Fluid, *Multiphase Science and Technology*, **5**, 239-361.
9. Pauchon, C. and Banerjee, S., 1988, Interphase Momentum Interaction Effects in the Averaged Multifield Model. Part II: Kinematic Waves and Interfacial Drag in Bubbly Flows. *Int. J. Multiphase Flow*, **14**, 253-264.