MODELING TURBULENCE NON-ISOTROPY IN DISPERSED PARTICLE/LIQUID FLOWS

A. Alajbegovic[a, 1], D.A. Drew[b], and, R.T. Lahey, Jr.[b]
[a] *Advanced Simulation Technologies, AVL Powertrain Engineering, Inc.*
47519 Halyard Dr., Plymouth, MI 48170-2438
[b] *Center for Multiphase Research, Rensselaer Polytechnic Institute*
Troy, New York, 12180-3590, USA

Abstract – Turbulence plays a very important role for the distribution of particles in dispersed particle/liquid flows. Single particle motion is affected by the level of the surrounding turbulence field and the size of turbulent eddies. Both parameters are customarily included in the development of dispersion models. What is usually neglected is the non-isotropy of the turbulent field. It is well known from the available experimental data that turbulence field is seldom isotropic. Even in simple geometries, like pipe flow, turbulence is highly non-isotropic with the largest component in the flow direction. The non-isotropy affects the motion of particles. The modeling of turbulence non-isotropy requires the solution for the complete Reynolds stress, which means solving six additional partial differential equations instead of the customary solution of the turbulence kinetic energy equation. This represents considerable computational effort. In the present study an algebraic stress model for the non-isotropic velocity fluctuations for both the continuous (liquid) and dispersed (solid) phase is presented. This model requires considerably less computational resources than solving the complete set of Reynolds stress equations. The model has been tested using a computational fluid dynamics (CFD) code. Simulations were done for solid particles of different densities – positive, negative, and neutral buoyant particles. The results were compared against experimental data. Good agreement with the computed results was observed.

SUMMARY OF TURBULENCE NON-ISOTROPY MODELING

The continuous fluid phase velocity fluctuation can be decomposed into a particle-induced (*PI*) and shear-induced (*SI*) part (Sato & Sekoguchi, 1975; Theofanus & Sullivan, 1982):

$$\mathbf{v}'_c = \mathbf{v}'^{PI}_c + \mathbf{v}'^{SI}_c \qquad (1)$$

Lance & Bataille (1991) performed experiments in bubbly flows with grid generated turbulence and found that the above decomposition is valid for dilute dispersions, as is the case in the present study.

Particle-induced velocity fluctuations can be estimated using the inviscid flow assumption for the flow around each particle. Applying cell model averaging yields the Reynolds stress tensor (Nigmatulin, 1979; Arnold *et al.*, 1989; Park, 1992) in the following simple form:

$$\overline{\tau}_c^{\mathrm{Re},PI} = \rho_c \frac{\alpha_d}{\alpha_c}\left[a_l \overline{\mathbf{v}}_r \overline{\mathbf{v}}_r + b_l \left(\overline{\mathbf{v}}_r \cdot \overline{\mathbf{v}}_r\right)\mathbf{I}\right] \qquad (2)$$

where, $a_l = -1/20$, $b_l = -3/20$. As implied before, the total Reynolds stress of the continuous phase can be written as the sum of particle-induced and shear-induced parts:

$$\overline{\tau}_c^{\mathrm{Re}} = \overline{\tau}_c^{\mathrm{Re},PI} + \overline{\tau}_c^{\mathrm{Re},SI} \qquad (3)$$

The continuous phase shear-induced Reynolds stress, $\tau_c^{\mathrm{Re},SI}$, was modeled using an algebraic stress model (Demuren & Rodi, 1984). For the dispersed phase Reynolds stress we make the classical assumption that:

$$\frac{\overline{\mathbf{v}'_i \mathbf{v}'_j}}{\overline{\mathrm{k}}} \cong Constant \qquad (4)$$

Based on this assumption and using both turbulence kinetic energy equation and Reynolds stress equation for the dispersed phase, it is easy to prove that:

$$\frac{D\alpha_d \rho_d \overline{\mathbf{v}'_{d,i}\mathbf{v}'_{d,j}}}{Dt} \cong \frac{\overline{\mathbf{v}'_{d,i}\mathbf{v}'_{d,j}}}{\overline{\mathrm{k}}_d}\frac{D\alpha_d \rho_d \overline{\mathrm{k}}_d}{Dt} \qquad (5)$$

1 email: ales@avlna.com

and,

$$\frac{\partial C_\alpha \overline{v'_{d,i} v'_{d,j} v'_{d,l}}}{\partial x_l} \cong$$

$$\frac{\overline{v'_{d,i} v'_{d,j}}}{\bar{k}_d} \frac{\partial}{\partial x_l} \left(C_\alpha C_d \frac{\Lambda_d}{\bar{k}_d^{\frac{1}{2}}} \overline{v'_{d,l} v'_{d,m}} \frac{\partial \bar{k}_d}{\partial x_m} \right) = \tag{6}$$

$$\frac{\overline{v'_{d,i} v'_{d,j}}}{\bar{k}_d} \frac{\partial C_\alpha \overline{k_d v'_{d,l}}}{\partial x_l}$$

where the contribution due to particle collisions is contained in:

$$C_\alpha = \rho_d \alpha_d \left(1 + \tfrac{16}{5} g(\mathbf{x}) \alpha_d\right) \tag{7}$$

We note that the triple correlation term is modeled by analogy with turbulent single-phase flow. Both equations may be used to rewrite the dispersed phase Reynolds stress equation as,

$$-\alpha_d \rho_d \left(\overline{v'_{d,l} v'_{d,i}} \frac{\partial \bar{v}_{d,j}}{\partial x_l} + \overline{v'_{d,l} v'_{d,j}} \frac{\partial \bar{v}_{d,i}}{\partial x_l} \right)$$

$$- K_{ij} + W^T_{d,ji} + W^T_{d,ij} = \tag{8}$$

$$\frac{\overline{v'_{d,i} v'_{d,j}}}{\bar{k}_d} \left\{ -\alpha_d \rho_d \overline{v'_{d,i} v'_{d,l}} \frac{\partial \bar{v}_{d,i}}{\partial x_l} - K_l + W^T_d \right\}$$

where the contribution due to interparticle collisions is:

$$K_{ij} = \frac{\partial}{\partial x_l} \left(\tfrac{8}{5} \rho_d g \alpha_d^2 \left(\overline{v'_{d,j} k_d} \delta_{il} + \overline{v'_{d,i} k_d} \delta_{jl} \right) \right) \tag{9}$$

$$K_l = \frac{\partial}{\partial x_l} \left(\tfrac{8}{5} \rho_d g \alpha_d^2 \overline{v'_{d,l} k_d} \right) \tag{10}$$

These two terms can be neglected when the dispersed phase volume fraction is small and there are not many collisions between the particles. Equation (8) can be used to express the dispersed phase's Reynolds stress as:

$$\overline{v'_{d,i} v'_{d,j}} =$$

$$\frac{\bar{k}_d \overline{v'_{c,i} v'_{c,j}} - \frac{\bar{k}_d \tau_R}{2 \alpha_d \rho_d C_1^W} \left(\alpha_d \rho_d \left[\overline{v'_{d,l} v'_{d,i}} \frac{\partial \bar{v}_{d,j}}{\partial x_l} + \overline{v'_{d,l} v'_{d,j}} \frac{\partial \bar{v}_{d,i}}{\partial x_l} \right] + K_{ij} \right)}{\bar{k}_c - \frac{\tau_R}{2 \alpha_d \rho_d C_1^W} \left(\alpha_d \rho_d \overline{v'_{d,l} v'_{d,i}} \frac{\partial \bar{v}_{d,i}}{\partial x_l} + K_l \right)} \tag{11}$$

Interestingly, in the case of negligible collisions and small gradients in the dispersed phase's mean velocity, the above equation reduces to,

$$\overline{v'_{d,r} v'_{d,r}} = C_R \frac{\bar{k}_d}{\bar{k}_c} \overline{v'_{c,r} v'_{c,r}} \tag{12}$$

where the coefficient, C_R, equals to 0.7 for the lateral normal stresses and 1.0 for the shear and axial Reynolds stress.

CONCLUSIONS

The resultant models were implemented into a CFD code. The flow parameters were predicted over a wide range of flow conditions and particle densities as can be seen in Figures 1, 2 and 3. Overall, the two-fluid model resulted in good agreement with the experimental data. It appears that two-fluid models are inherently capable of predicting multidimensional phase distribution phenomena in turbulent multiphase flows.

Acknowledgments - The authors wish to acknowledge the financial support given this research by the National Science Foundation (NSF).

REFERENCES

Alajbegovic, A. Assad, F. Bonetto, R.T. Lahey, Jr., *Int. J. Multiphase Flow*, **20**, 453 (1994).

G.S. Arnold, D.A. Drew, R.T. Lahey Jr., *Chem. Engin. Comm.*, **86**, 43 (1989).

O.A. Demuren, W. Rodi, *J. Fluid Mechanics*, **140**, 189 (1984).

M. Lance, J. Bataille, *J. Fluid Mechanics*, **222**, 95 (1991).

R.I. Nigmatulin, *Int. J. Multiphase Flow*, **5**, 353 (1979).

J-W. Park, Ph.D. Thesis, Rensselaer Polytechnic Institute, Troy, New York (1992).

Y. Sato, K. Sekoguchi, *Int. J. Multiphase Flow*, **2**, 79 (1975).

T.G. Theofanus, J.P. Sullivan, *J. Fluid Mechanics*, **116**, 343 (1982).

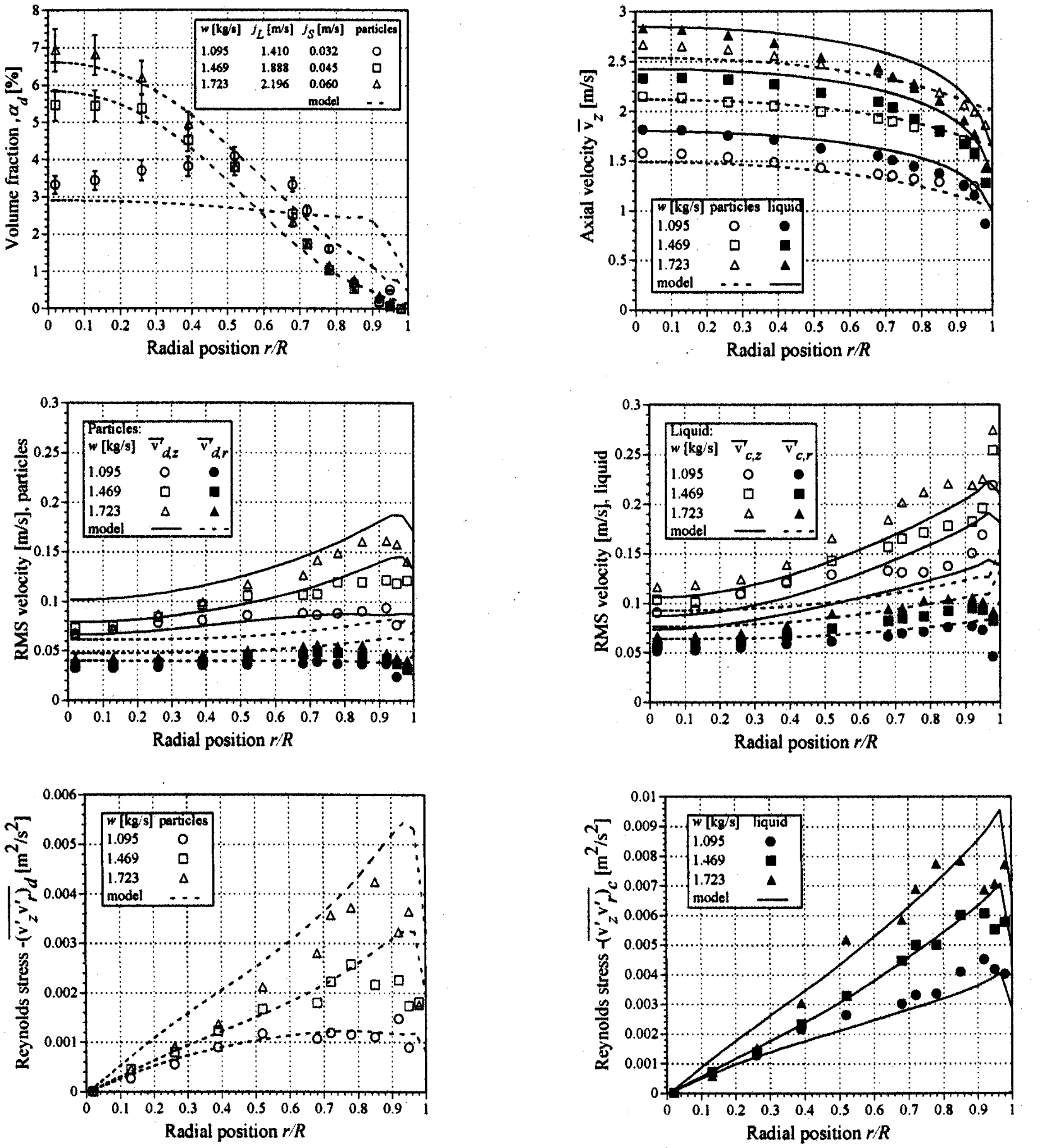

Figure 1: Comparison between the predicted and measured parameters for flow with negative buoyant spheres (experimental data by Alajbegovic et al., 1994).

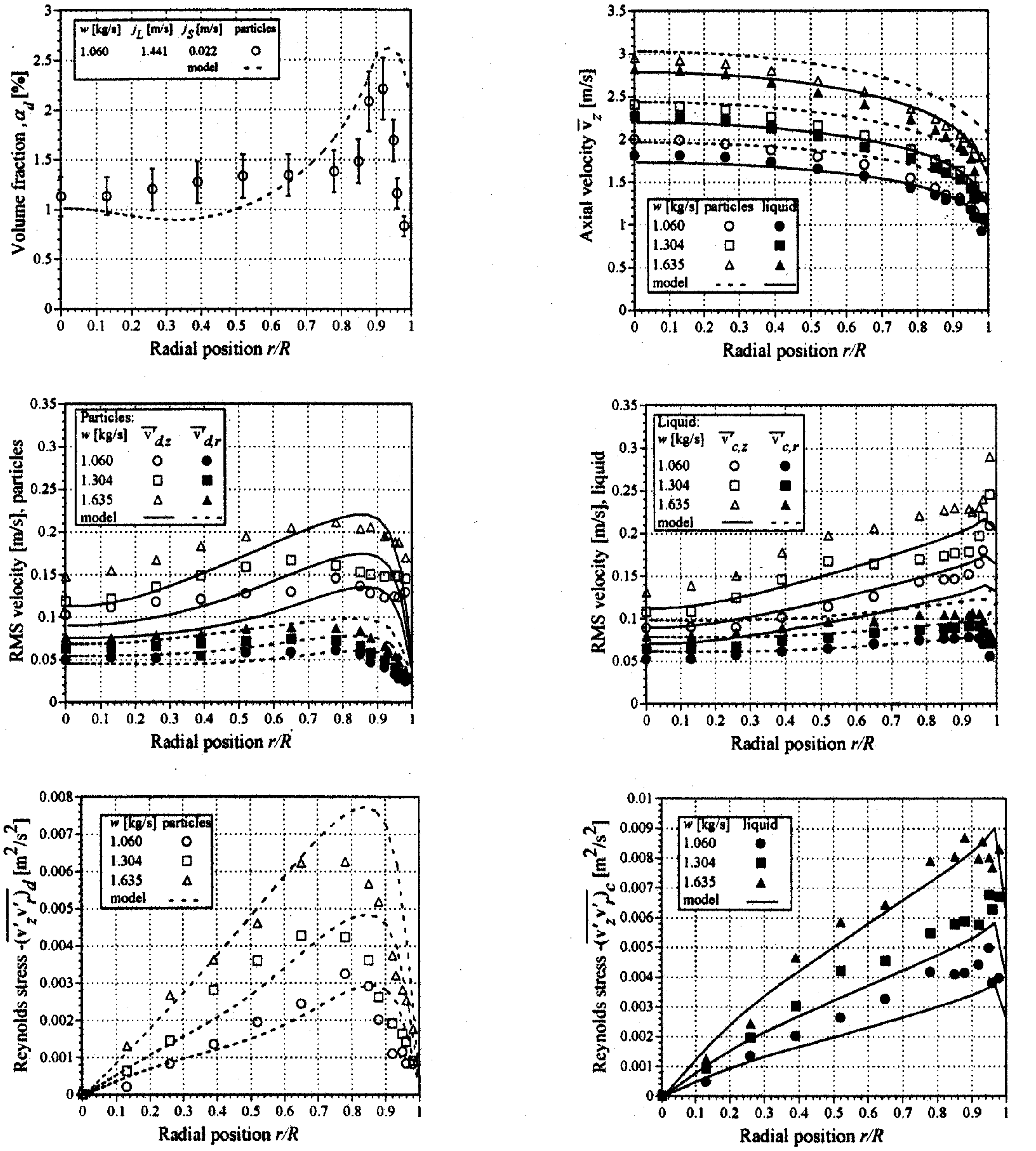

Figure 2: Comparison between the predicted and measured parameters for flow with positive buoyant spheres (experimental data by Alajbegovic et al., 1994).

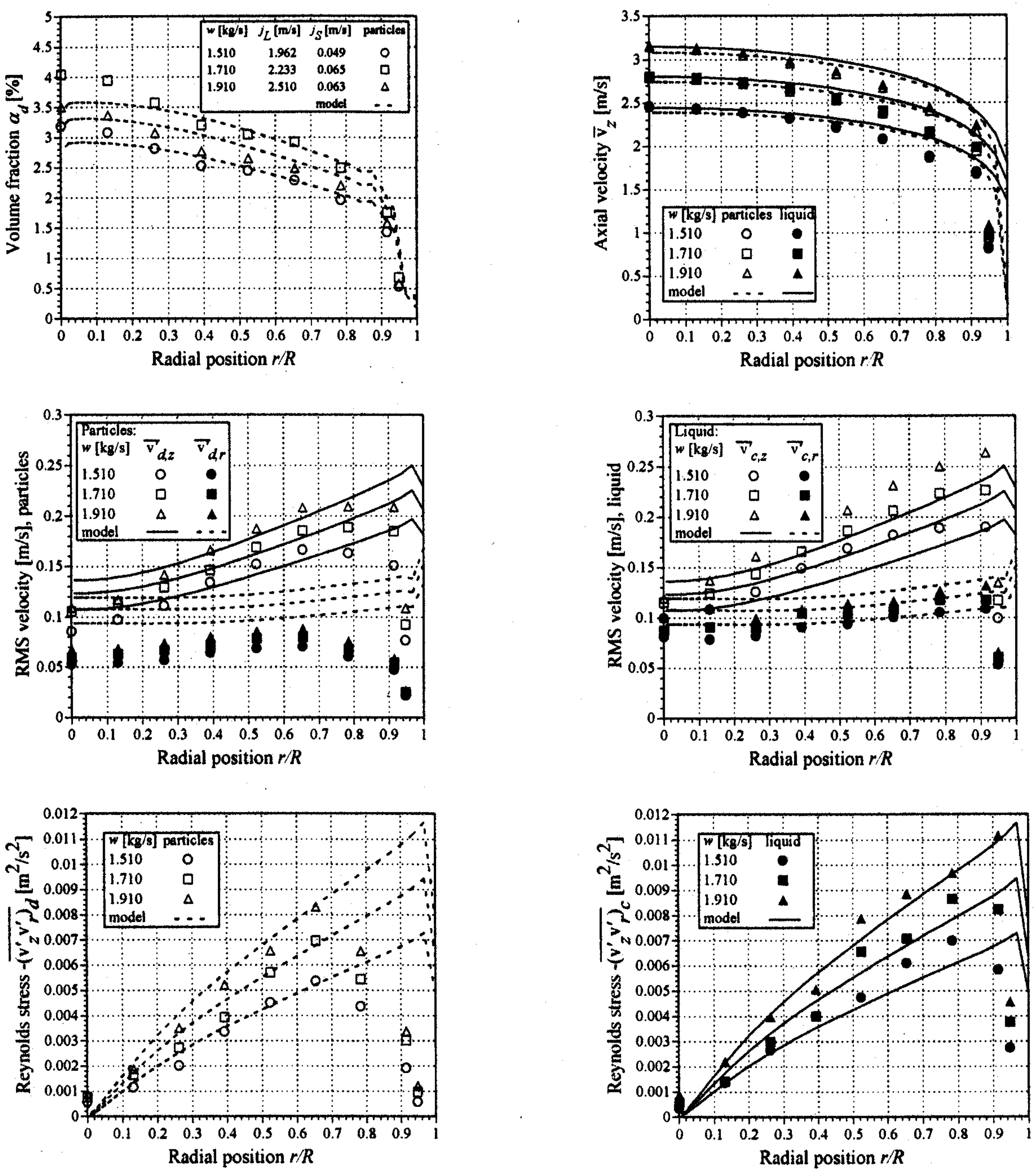

Figure 3: Comparison between the predicted and measured parameters for flow with neutral buoyant spheres (experimental data by Alajbegovic et al., 1994).