A STATISTICAL MODEL FOR PREDICTING THE COLLISION RATE OF PARTICLES IN ISOTROPIC TURBULENCE

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The objective of the paper is to present an analytical statistical model for predicting the collision rate of particles suspended in a turbulent fluid. This model allows for both the collisions induced by turbulence and the effects caused by mean relative velocities between particles. The model under development is compared with the results of numerical simulations for turbulent suspensions of identical particles as well as a binary mixture of particles having different densities.

INTRODUCTION

The rate of coagulation due to collisions of solid particles or liquid droplets in multiphase turbulent media is of importance in many meteorological and industrial processes. Examples include precipitation of aerosols, separation in cyclones, fluidized beds, spray combustion, vapour-droplet flows in condensing steam nozzles and turbine cascades, and so on. Because of practical importance, by now numerous theoretical studies of the collision rate induced by turbulence have been performed. Relatively simple closed solutions to this problem may be apparently derived only in homogeneous isotropic turbulence. Two solutions are most familiar in the literature, corresponding to the limiting cases of non-inertial and high-inertial particles. The first approach is valid for fine particles whose response time is less than the time microscale of turbulence, and, consequently, which completely follow the fluid turbulence[1]. The second approach relates to particles whose response time is much greater than the integral time scale of turbulence, and, hence, the relative motion of colliding particles is completely uncorrelated and similar to the chaotic motion of molecules in the kinetic theory of rarefied gases[2]. In this paper, we present an analytical statistical model for predicting the collision rate that is valid over the entire range of particle inertia (from zero-inertia to high-inertia case). Moreover, this model takes into account the contribution of the mean relative velocity between particles (inter-particle drift) to the collision rate. For validating the model under development, results from direct numerical simulation (DNS) and large eddy simulation (LES) are used for particles being dispersed in a turbulent fluid.

INTER-PARTICLE COLLISIONS UNDUCED BY TURBULENCE

The collision rate (kernel) is defined on the basis of the so-called spherical formulation in terms of the average radial component of the relative velocity of colliding particles. Then the assumption is made that the relative velocities follow the Gaussian probability distribution. The fluctuating velocities of the fluid and particles, as well as velocity gradients are also presumed to be Gaussian. These probability distributions allow for the correlation between the velocities of two colliding particles as a result of particle-turbulence interaction. The correlation effect is very important when the particle response time is in order of the integral time scale of turbulence or lower. To describe the particle-turbulence interaction, a two-scale velocity auto-correlation function, characterizing by micro- and macroscales of turbulence, is used[3]. In this way we obtain an analytical expression for the collision rate of two particles. When the mean relative velocity between particles (inter-particle drift) is absent, and therefore collisions are caused only by interactions between particles and fluid turbulence, the collision rate takes the following form

$$\beta_t = \left(8\pi\langle w_r'^2\rangle\right)^{1/2}\sigma^2 = \sqrt{\frac{8\pi}{3}}\sigma^2\left[2\left(k_{p1} + k_{p2} - 2R_{12}k_{p1}^{1/2}k_{p2}^{1/2}\right) + \frac{\varepsilon}{5\nu}\sigma^2\right]^{1/2} \qquad (1)$$

Here $\sigma = r_1 + r_2$ is the collision radius where r_1 and r_2 are the radii of two colliding particles, $w_r = \mathbf{w}\cdot\mathbf{r}$ designates the radial component of the relative velocity $\mathbf{w}$, $\mathbf{r}$ denotes the separation vector directed from the center of the first particle to the center of the second one, k_{p1} and k_{p2} are the turbulent kinetic energies of particles, ε is the fluid turbulent energy dissipation rate, ν is the kinematic viscosity, and R_{12} denotes the correlation coefficient that reflects a response of particles to small-scale and large-scale turbulent fluctuations. In the small and large particle limits, this expression contracts, respectively, into the relationships for the collision kernel derived by Saffman and Turner[1] and Abrahamson[2].

To verify the model developed for determining the turbulence-induced collision rate, comparisons between model predictions and DNS computations[4-6] have been carried out for identical particles suspended in both isotropic and channel flows. Some results of these comparisons for isotropic turbulence are presented in Figures 1 and 2. Figure 1 shows the normalized collision rate as a function of the particle response time, τ_p, related to the Kolmogorov time microscale, τ_k, for low-inertial particles. As is seen, for sufficiently small particles the collision rate rises with increasing particle inertia, starting from the Saffman-Turner relationship for non-inertial particles. In accordance with the numerical simulations, the model predicts an enhancement of the collision rate as the ratio of the particle size to the Kolmogorov length scale, η, decreases and the Reynolds number based on the Taylor length microscale, Re_λ, increases.

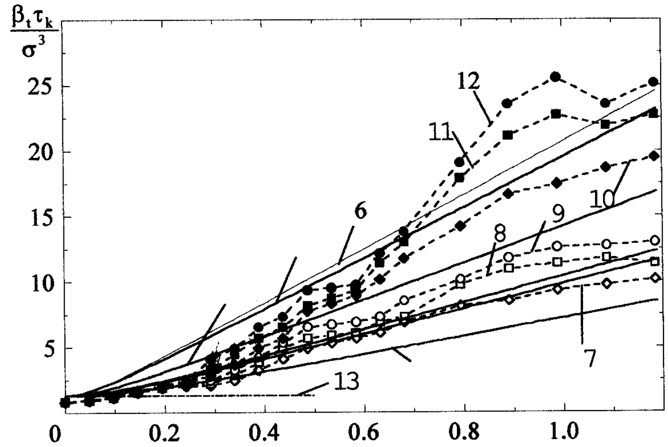

Figure 2 demonstrates the dependencies of the collision rate on the particle response time over a wide range of particle inertia. The computations displayed in Figure 2 were performed for various collision counting schemes which brought into slightly different numerical collision kernels. It is evident that both the predictions and numerical simulations follow the same trend and indicate in particular a pronounced maximum. The maximum reflects a reduction in the correlation between particle velocities at collision as the particle response time increases. This causes an increase in the relative velocity between two particles and a concomitant gain in the collision rate. The subsequent fall in the collision kernel with τ_p results from a decrease in the particle turbulent kinetic energy since the particles became less responsive to the turbulent fluctuations of the fluid.

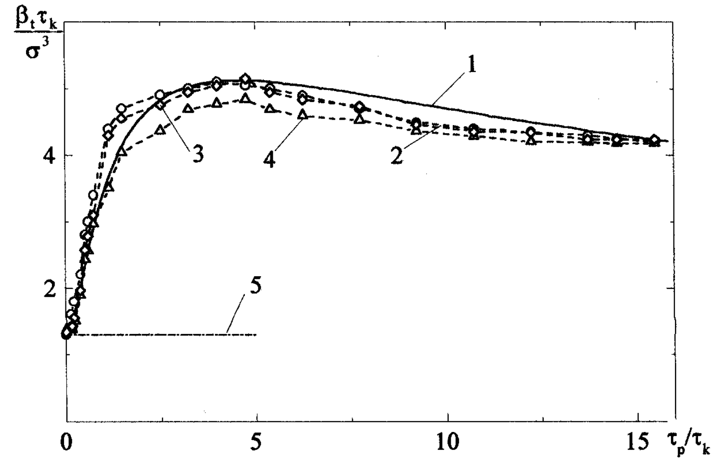

Figure 2. Effect of particle inertia on the collision rate (Re_λ =24 and σ / η =1.78): 1 - prediction; 2,3,4 - simulations[5]; 5 - Saffman and Turner[1]

THE EFFECT OF INTER-PARTICLE DRIFT

Further let us consider collisions caused by two simultaneous mechanisms: the fluid-phase turbulence and the mean relative velocity between particles due to, e.g., gravity. In this case, to determine the collision rate, it is required to perform an averaging over both the random distribution of the radial relative velocity, w_r, and the angle which characterizes a spatial orientation of $\mathbf{w}$ in reference to the separation vector, $\mathbf{r}$. By this means we derive the following expression for the collision rate

$$\beta = (8\pi\langle w_r'^2\rangle)^{1/2}\sigma^2\left[\frac{\exp(-\Sigma^2)}{2} + \frac{\pi^{1/2}}{2}\left(\Sigma + \frac{1}{2\Sigma}\right)\operatorname{erf}\Sigma\right] \tag{2}$$

Here $\Sigma = W_g / (2\langle w_r'^2\rangle)^{1/2}$ characterizes the gravity effect with respect to the turbulence-induced collision rate, $W_g = |\tau_{p1} - \tau_{p2}| g$ stands for the mean relative velocity between two particles, and g is the gravity acceleration. In the case of no gravity ($\Sigma = 0$), equation (2) reduces to (1). When $\Sigma \to \infty$ (2) goes over into the familiar expression for the collision kernel due to gravity

$$\beta_g = \pi\sigma^2 W_g$$

To examine equation (2), a binary mixture of two classes of particles (with equal sizes but different densities) suspended in an homogeneous isotropic turbulent field generated by LES[7] was considered. The objective of this analysis was to verify the predictions of the mutual effect of turbulence-induced and inter-particle drift collision mechanisms. Predicted collision frequencies were found to be quite well consistent with LES computations for all the cases considered by Gourdel et al.[7]

SUMMARY

An analytical statistical model that describes inter-particle collisions in homogeneous isotropic turbulence is developed. This model predicts the collision rate as a function of turbulence parameters and particle properties. The model proposed is correct over the entire range of particle inertia (from zero-inertia to high-inertia case). Moreover, this model takes into consideration the effect of inter-particle drift due to, e.g., gravity.

On the basis of comparisons performed with DNS and LES computations, the conclusion is drawn that the model developed can plausibly predict the collision rate due to both the particle-turbulence interaction and the inter-particle drift effect.

ACNOWLEDGEMENT

This work was partially supported by the Russian Foundation of Basic Investigations (grant 00-02-17254).

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