DETERMINATION OF THE COEFFICIENT OF TRANSVERSE DISPERSION IN BEDS OF SAND FOR A WIDE RANGE OF TEMPERATURES (50 < Sc < 1200)

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ABSTRACT

The more widely used techniques for the measurement of transverse dispersion are the continuous point source and the instantaneous finite source methods, which rely on the injection of a tracer in the flowing liquid, followed by tracer detection at several points, downstream of the injection point.

Our group (see[1]) developed a new experimental technique, based on the measurement of the rate of dissolution of planar or cylindrical surfaces, buried in the bed of inert particles and aligned with the flow direction. This alternative technique is simple to use, allows the determination of the coefficient of transverse dispersion (D_T) in packed beds over a wide range of flow rates, and it is easily adaptable to working over a range of temperatures above ambient, as shown in our paper[2, 3]. This is a considerable advantage, given the extreme lack of data on the effect of the Schmidt number on the dispersion coefficient; indeed, the vast majority of data available at present have been obtained in experiments with either water or air, at ambient temperature.

It is important to bear in mind that, to the authors' knowledge, no previous published reports are available of measurements of D_T for $2 < Sc < 540$, and we have obtained more than twice the number of experimental values of the coefficient of transverse dispersion than exist published.

The experiments were performed at each of eleven temperatures, in steps of 5 K in the range 293 K to 308 K and in steps of 10 K in the range 313 K to 373 K, and values of D_T were measured at thirteen different flow rates for each temperature. Having worked with five batches of silica sand, with average particle sizes of 0.219 mm to 0.496 mm, we obtained over four hundred values of D_T for a wide range of Reynolds number ($Re = Ud\rho/\mu$) between 0.01 and 3.5. Repetition of several runs showed that the values of D_T obtained were reproducible to within ±5 %.

We briefly describe the theoretical approach of Coelho and Guedes de Carvalho[1] (1988), who considered the problem of mass transfer from a flat surface. A final result for the average mass transfer coefficient, based on the slab surface area, is then

$$k = \varepsilon \left(\frac{4 D_T}{\pi L / u} \right)^{1/2} \qquad (1)$$

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where ε is the uniform bed voidage, L is the length of the soluble slab and u is the interstitial velocity. Experiments performed by Coelho and Guedes de Carvalho[1] (1988) with a wide range of slab lengths, both with benzoic acid in water and with naphthalene in air, confirm the general validity of the above theory, provided that the approximate criterion

$$\frac{L}{d} \gtrsim 0.62\left(\frac{ud}{D_m}\right) \tag{2}$$

is observed, where d is the size of the inert particles making up the bed and D_m is the molecular diffusion coefficient.

In terms of construction of equipment, it is normally easier to perform packed bed experiments inside cylindrical tubes, rather than in prismatic columns. Figure 1 sketches the test column used in our experiments, where the dissolving cylinder is tightly assembled between two rods of stainless steel and the set is buried in the packed bed, along its axis.

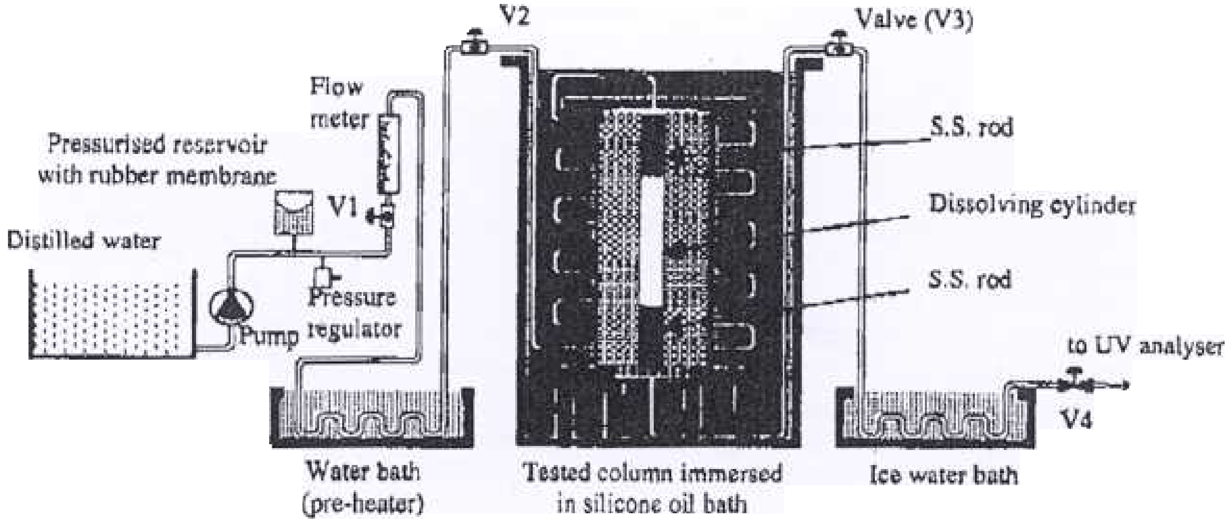

Figure 1 – Diagram of experimental set up.

Several hundred data points were obtained with the above rig and before attempting a more general correlation of the data it may be interesting to consider the separate influence of the various parameters on the value of D_T, and to this end it is important to know how the value of the relevant physical properties depends on the operating temperature.

One first aspect to be considered, as a check on the experimental method, is the influence of the length of the test cylinder (L) on the measured value of D_T. In reality, the two variables are independent, and if our experimental method is valid, values of the dispersion coefficient measured with different lengths of cylinder, under otherwise similar conditions, should be equal, within the reproducibility limits. The values of D_T plotted in Figure 2 are typical and they illustrate the independence between the measured values of that parameter and the length of the soluble cylinders used in the experiment, provided that the criterion given by (2) is satisfied, as required.
The variation of D_T with liquid velocity is illustrated in Figure 3 at two temperatures. An increase in temperature is seen to lead to an increase in the value of the dispersion coefficient, the effect being more pronounced at the lower velocities. Finally, the effect on the dispersion coefficient, of a distribution of particle sizes within the bed, may be assessed from our data. In particular, lot D was

prepared by carefully blending lots B and E in a proportion of 1:1 (by weight). In Figure 4, dispersion data obtained with the mixed lot are seen to fall in between the data for the original separate lots, as might be expected.

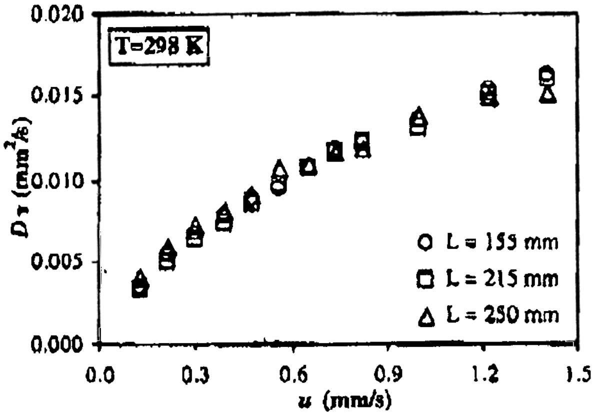

Figure 2 – Variation of D_T with u for different lengths of test cylinder.

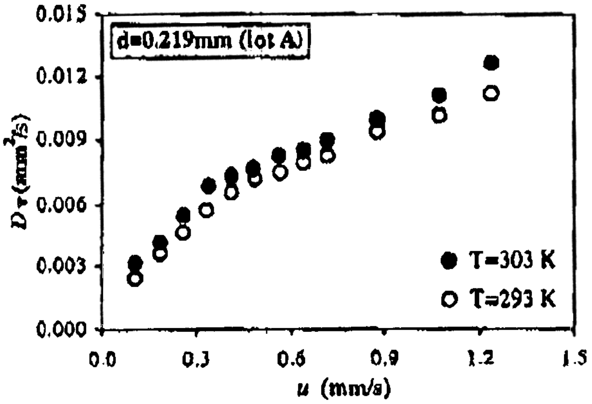

Figure 3 – Effect of temperature on variation of D_T with u.

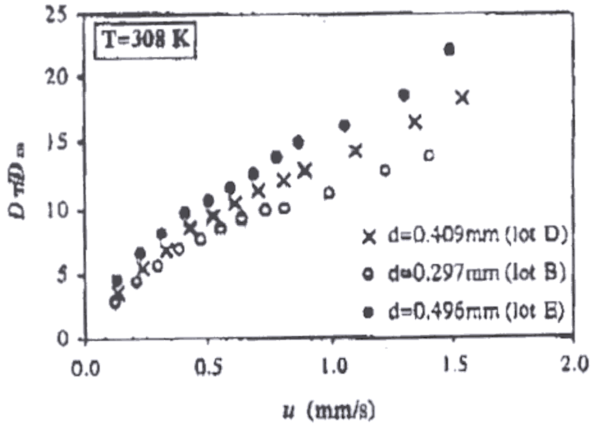

Figure 4 – Effect of particle size on the variation of D_T / D_m with u.

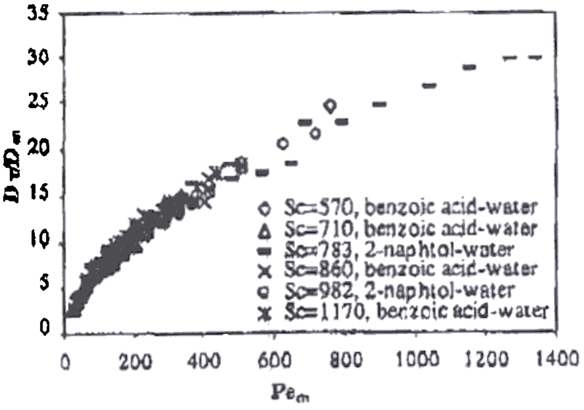

Figure 5 – Variation of D_T / D_m with Pe_m showing independence of Sc, for Sc ≥ 570.

In a first and long series of experiments, only cylinders of benzoic acid were used, with Reynolds numbers up to 0.24 and temperatures up to 308 K, to obtain dispersion data needed in the analysis of mass transfer around buried spheres (Guedes de Carvalho and Delgado[4], 1999). The system 2-naphtol-water was introduced when it was decided to work at higher temperatures, and simultaneously it was decided to extend the study to the maximum Peclet number ($Pe_m = ud /D_m$) compatible with the criterion given by (2) in our rig.

Plots of the coefficient of transverse dispersion (divided by the coefficient of molecular diffusion) *vs.* the Peclet number, bring into evidence the influence of Sc on transverse dispersion. As Sc is decreased, the values of D_T / D_m gradually approach the line corresponding to "gas behaviour" (i.e. Sc = 1), which is known to be well approximated by the equation

$$\frac{D_T}{D_m}=\frac{1}{\tau}+\frac{1}{12}\frac{ud}{D_m} \qquad (3)$$

In the analysis of the data collected, it was found that for Sc ≥ 550, plots of D_T / D_m *vs.* Pe_m fall approximately on a single line which corresponds to

$$\frac{D_T}{D_m} = -1.48 \times 10^{-5}\, Pe_m^{\,2} + \frac{0.51}{12} Pe_m + \frac{1}{\tau} \qquad Sc \geq 550$$

and it is believed to represent the lower limit for D_T / D_m, as Sc is increased, since experiments with values of Sc between 550 and 1476 all seem to plot along this line. For lower values of Sc, the values of D_T / D_m depend both on Pe_m and Sc and the dashed lines in Figure 6 correspond to equations of the form

$$\frac{D_T}{D_m} = \left(9.7 \times 10^{-11}\, Sc^2 - 8.4 \times 10^{-8}\, Sc + 5.6 \times 10^{-7}\right) Pe_m^{\,2} +$$
$$+ \frac{1}{12}\left(1 - 8.3 \times 10^{-4}\, Sc\right) Pe_m + \frac{1}{\tau} \qquad Sc < 550$$

It is important to bear in mind that eqns. (4) and (5) are only recommended for $Pe_m < 1400$, the upper limit for Pe_m in our experiments. Above that limit, those two equations were not tested and they may prove very inadequate, in particular since they do not have the correct asymptotic behaviour, for $Pe_m \rightarrow \infty$.

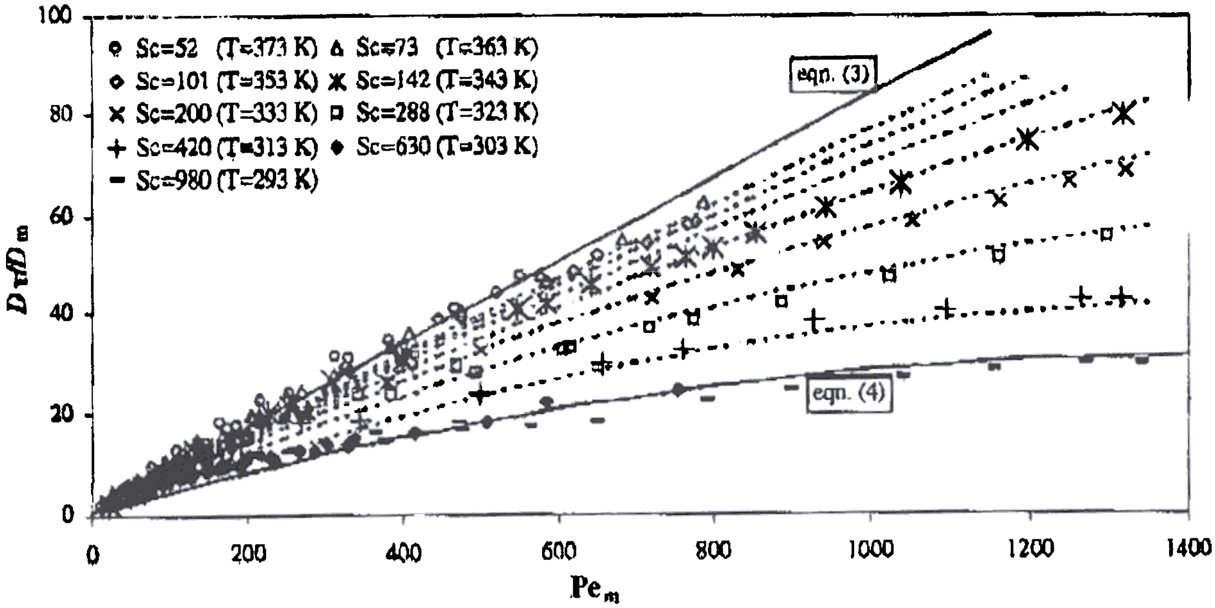

Figure 6 – Plot of experimental data from this work

In Figure 7 the experimental findings of other authors are shown, again in a plot of D_T / D_m *vs.* Pe_m. The solid lines corresponding to eqns. (3) and (4) are again represented and so are two dashed lines corresponding to a correlation proposed by Gunn[5] (1969). All the experimental points shown are for Sc ≥ 540 and therefore we would expect them to fall near the line representing eqn. (4). The greatest deviations are observed for the points of Grane and Gardner[6] (1961), but their scatter is pronounced, and the two points by Harleman and Rumer[7] (1963), for high Pe_m. All the other points seem to be well represented by eqn. (4).

Comparison of the predictions of the correlation proposed by Gunn[5] (1969), in the range $52 < Sc < 980$ shows the inadequacy of that correlation.

In the present state of knowledge we can recommended the use of equation (4) for $Sc > 550$ and eqn. (5) for $Sc < 550$, both only for $Pe_m < 1400$.

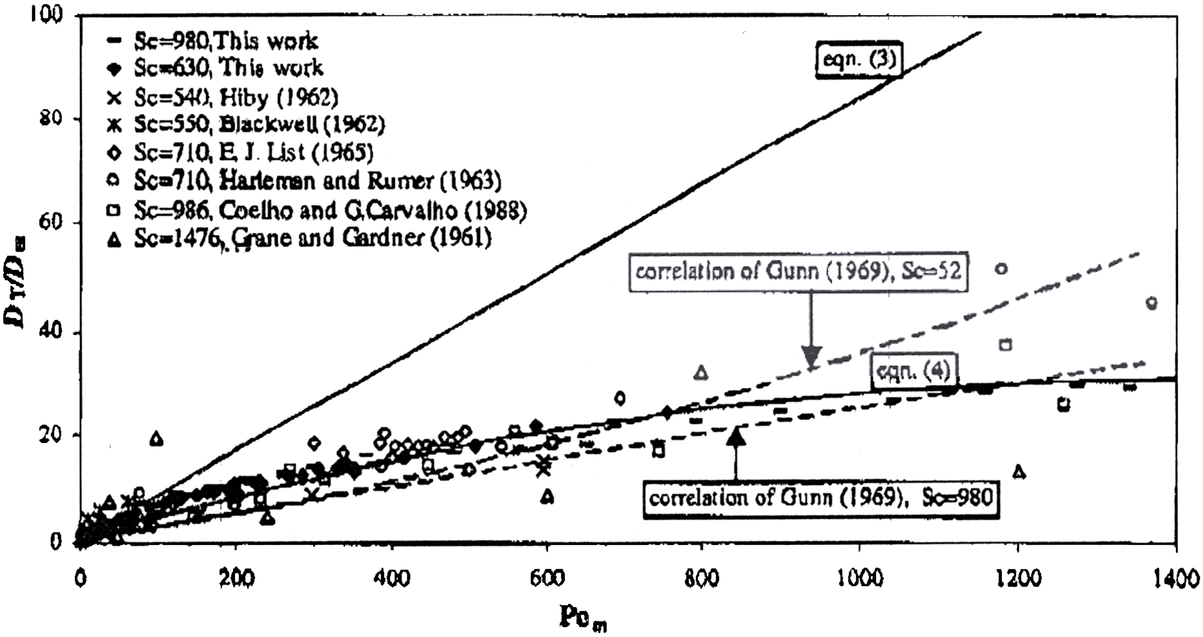

Figure 7 – Plot of published data for $Sc > 540$.

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