FLAPPING MOTIONS OF PLANAR LIQUID SHEETS INDUCED BY GASEOUS HEAT AND MASS TRANSFER

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Two-dimensional liquid sheets falling under gravity are employed to deposit a thin, uniform film on a moving solid substrate in coating processes. In this paper, the fluid dynamics of planar liquid membranes and sheets such as that shown schematically in Figure 1 is analyzed. Liquid membranes are liquid sheets of zero thickness that undergo stretching as a consequence of the gravitational pull, pressure differences across the membrane and surface tension. Moreover, the pressure of the gases on the liquid is not constant along the membrane due to the recirculating flow pattern created in the gases by entrainment on both sides of the membrane. In fact, at high Reynolds numbers such as the ones considered here, the gases on both sides of the membrane have enclosed streamlines characterized by the Prandtl-Batchelor theorem and boundary layers along the membrane and solid walls where vorticity is generated. In this paper, the rheology of the liquid membrane, i.e., surface viscosity, is neglected, and, by analyzing the deformation of liquid elements, we have obtained the conservation laws for the mass and momentum per unit length of the liquid membrane as well as a kinematic boundary condition which depend on time and the coordinate along the membrane, surface tension, gravitational pull, pressure differences across the membrane and friction forces exerted by the gases on the liquid. These one-dimensional equations are nonlinearly coupled to the fluid dynamics of the gases that surround the membrane, and these gases have been assumed to be ideal and Newtonian. The fluid dynamics of the gases that surround the liquid has been analyzed by means of two different methods. The first technique is valid at high Reynolds numbers and divides the volume occupied by the gases into an inviscid albeit rotational region core region with closed streamlines and boundary layers where vorticity is generated. By matching the vorticity at an outer streamline in the boundary layer with that of the inviscid core we end up with a problem which is elliptic in the core and parabolic in the boundary layers. In the second technique, the fluid dynamics of the gases that surround the liquid membrane have been determined by solving numerically the two-dimensional Navier-Stokes equations in Cartesian coordinates subject to the kinematic and dynamic boundary conditions at the solid walls and at the liquid membrane.

The equations governing the fluid dynamics of inviscid liquid membranes can be written as

$$v = h_t + uh_x, \tag{1}$$

$$m(u_t + uu_x) = mg - (\Delta p + 2\sigma \cos^3\theta h_{xx})\sin\theta - (f_1 + f_2)\cos\theta, \tag{2}$$

$$m(v_t + uv_x) = (\Delta p + 2\sigma \cos^3\theta h_{xx})\cos\theta - (f_1 + f_2)\sin\theta, \tag{3}$$

where $\tan\theta = h_x$, m is the membrane's mass per unit length, u and v are the liquid's velocity components along the x and y axes, respectively, g is the gravitational acceleration, σ is the surface tension, Δp is the pressure difference across the membrane, f_1 and f_2 denote the friction forces exerted by the gases on the left and right of the liquid membrane, respectively, x and y are the vertical and horizontal Cartesian coordinates, respectively, the subscripts denote partial differentiation, and h denotes the membrane's horizontal displacement or shape.

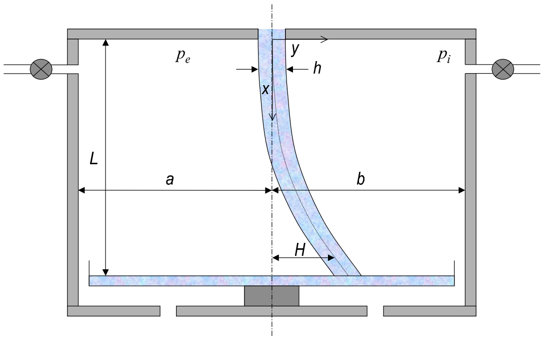

Figure 1: Schematic of a planar liquid sheet.

For steady flows, it is shown that planar liquid membranes may exhibit a removable singularity at the nozzle exit for a Weber number equal to one and downstream of the nozzle exit for Weber numbers less than one; for a Weber number equal to one, the liquid exits the nozzle with an angle $\theta(0)$ given by $\sin\theta(0) = \Delta p/mg$ and this value corresponds to $rank A = rank(A, \mathbf{b}) = 2$ where A is a matrix such that $A\mathbf{v}_x = \mathbf{b}$ and $\mathbf{v} = (u, v, h)^T$ and the superscript denotes transpose. The existence of this singularity has been determined by analyzing the singularities of the differential equations that govern the flow and by means of algebraic methods, and its removable character indicates that the angle with which the liquid leaves the nozzle does not coincide with the nozzle exit angle for Weber numbers less than or equal to one. Moreover, a linear stability analysis of inviscid, planar liquid membranes surrounded by dynamically passive gases indicates that that the perturbed displacement from the centerline is governed by the following second-order hyperbolic equation

$$(\partial_t + g\partial_x)^2 h' = (2\sigma g^2/\dot{V})(h'_U/U)_U, \tag{4}$$

where U is the steady state velocity component in the vertical direction, h' is the perturbed horizontal displacement of the liquid membrane, and $\dot{V}$ is the steady mass flow rate. This equation is valid provided that $g \neq 0$. For $g = 0$, the corresponding equation is

$$(\partial_t + U\partial_x)^2 h' = (2\sigma U/\dot{V})h'_{xx}, \tag{5}$$

which has travelling wave solutions in accord with available experimental data for planar liquid sheets. Similar results have been found when a von Karman-Pohlhausen technique is employed to determine the boundary layers in the gases and thus account for frictional effects on the liquid membrane. This technique yields

$$f_i = n\mu_i W(x)/\Delta_i(x), \tag{6}$$

where W denotes the liquid's velocity along the membrane, Δ_i is the boundary layer thickness, μ is the dynamic viscosity of the gases that surround the liquid, $i = 1, 2$ denotes the gases to the left and right of the liquid membrane, respectively,

$$\left(W^2\Delta\left((2n+1)^{-1} + T_1K(x) + T_2K^2(x)\right)\right)_x = -n\nu W/\delta, \tag{7}$$

$$T_1 = 4((2n+1)(2n+2)(2n+3))^{-1}, \qquad T_2 = 6T_1((2n+4)(2n+5))^{-1}, \tag{8}$$

$$K(x) = \left((\Delta^2/\nu)W_x - n(n-1)\right)/2, \tag{9}$$

$u_g = W(x)(1-\psi)^n(1+\psi^2K(x))$, $\psi = (y - h(x))/\Delta$, the subscript g denotes gas, and ν is the gas kinematic viscosity.

The dispersion relation for Eq. (5) is ($h' = A \exp i(kx - \omega t)$)

$$\omega = k\left(U \pm \left(2\sigma U/\dot{V}\right)^{\frac{1}{2}}\right), \tag{10}$$

which indicates that h' oscillates periodically in space and time.

It is also shown that, when $\Delta p + 2\sigma\cos^3\theta h_{xx} = 0$, the shape of the liquid membrane is an arc of circumference.

A long-wave analysis of inviscid, planar liquid sheets in dynamically passive surroundings has also been performed in order to determine the leading-order equations which govern their fluid dynamics and compare these equations with those of planar liquid membranes. Such an analysis is based on the assumptions that the sheet's horizontal displacement, H, is small compared with the axial distance from the nozzle exit to the substrate, L, and results in three partial differential equations for the axial velocity component of the liquid and the displacements of the two free surfaces of the sheet which, after one integration, result in a nonlinear, second-order ordinary differential equation for the sheet's thickness. This equation has been solved analytically (in closed form) in a few cases, and asymptotically for large and small values of x by means of the dominant balance method, and the results indicate that the sheet's centerline leading-order displacement is an arc of a parabola when $\Delta p \neq 0$ and a straight line when $\Delta p = 0$.

A long-wave or lubrication approximation has also been employed for liquid sheets whose thickness, h, is smaller than a characteristic horizontal displacement of the liquid and this ratio is, in turn, smaller than the ratio of a characteristic horizontal displacement of the liquid to the vertical distance between the nozzle exit and the substrate.

Numerical studies of the membrane's and sheet's leading-order equations have also been performed when the pressures of the gases on both sides of the membrane are time-dependent due to the periodic injection or extraction of these gases; in some of these studies, the absorption of gases by the flowing liquid sheet was also accounted for by solving a high Peclet number concentration equation in the liquid domain. Moreover, for small injection rates, the gases surrounding the liquid were treated as isothermal, whereas they were considered to behave adiabatically at high injection rates. These studies were aimed at determining the effects of time-dependent effects on the fluid dynamics of liquid membranes and sheets and on coating processes. The studies did not pose difficult computational problems when the fluid dynamics of these gases was analyzed as consisting of an inviscid core and boundary layers and the liquid did not impinge on the lateral walls shown in Figure 1. When the full, two-dimensional equations for the gases were solved numerically, the domains were first mapped into fixed unit squares because the physical domain was time-dependent due to the horizontal motion of the liquid sheet and the sheet's shape had to be determined as part of the numerical solution. When the liquid did not impinge on the lateral solid walls, Hermite interpolation was employed for grid generation, but the same technique required quite a lot of computational effort when the sheet hit the lateral walls and, for example, the gases on the left were compressed.

Some sample results corresponding to the absorption of gases by the liquid sheet are illustrated in Figures 2 and 3. These figures were obtained by solving numerically the equation for both the gas dissolved in the liquid under the assumption that the interfaces were clean and the interfacial gas concentrations were proportional to the partial pressures of the gases through Henry's solubility law, and the temperatures of both the liquid and the gases on both sides of the sheet. In these figures, We, Fr, Pe_M and Pe_T denote the Weber, Froude and mass and

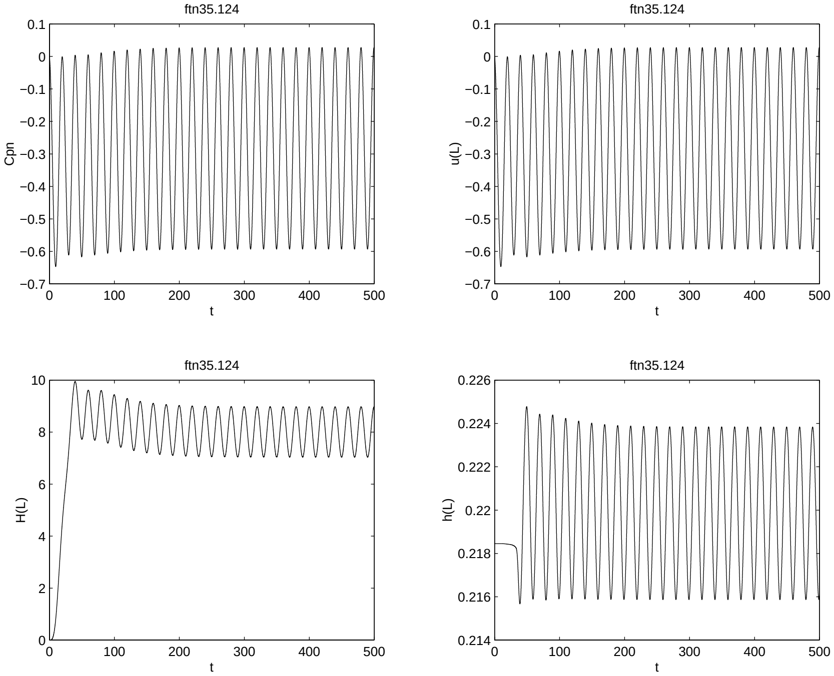

Figure 2: Pressure coefficient $C_{pn} = p_i/p_e - 1$ (top left), and sheet's axial velocity component (top right), horizontal displacement (bottom left) and thickness (bottom right) at $x = L$ as functions of time. ($Pe_M = 10^6$, $Pe_T = 10^7$, $We = 50$, $Fr = 10$, $\rho_l/\rho_g = 1000$, $C_{pl}/C_{vg} = 100$, $\gamma = 1.4$, $p_i(0)/p_e(0) = 1$, $T_i(0) = 1.5$, $T_e(0) = 1$, $T(0, r, t) = 1$, $c(0, r, t)/(S_i p_i) = 1$, $S_e/S_i = 1$, $S_e/R_g T_{ref} = 1$, $L = a = b = 100$, $\theta(0, t) = 0$).

thermal Peclet numbers, respectively, C_p and C_v are the specific heats at constant pressure and at constant volume, repectively, T is temperature, the subscripts ref, l and g refer to reference, liquid and gas, respectively, S is the solubility, R is the gas constant, and the subscripts e and i refer to the left and right sides of the liquid sheet, respectively, and the initial conditions correspond to hot gases on the right of the liquid sheet.

Figure 2 indicates that the displacement of the sheet's midline, $H(L)$, where L denotes the distance between the nozzle exit and the substrate increases from 0 to 8, i.e., the liquid sheet moves from the centerline on account of the heat and mass transfer between the gases and the liquid, and exhibits an oscillatory behaviour due to the compressibility of the gases that surround the liquid. The sheet's thickness and axial velocity component at $x = L$ also exbibit an oscillatory behaviour. On the other hand, the pressure coefficient $C_{pn} = p_i/p_e - 1$ decreases from its initial value of 0 to about a mean value equal to -0.3. Figure 3 indicates that the gas concentration on the right side of the sheet decreases as a function of time while the interfacial gas concentration on the left side of the sheet first decreases and then oscillates periodically about a mean value equal to zero. The gas absorption rate on the right side of the liquid sheet decreases monotonically due to the decrease in pressure associated with heat and mass transfer. Some of the gases absorbed at the right interface are convected downstream by the liquid but some diffusion occurs across the liquid sheet. However, for the case presented in Figures 2 and 3, the gas diffusion across the sheet is small due to the small binary diffusion coefficients of gases in liquids, i.e, the large mass Peclet number. Moreover, since the liquid emerged from the nozzle exit with some dissolved gases, some of this gas content flowed from the liquid to the gases on the right of the liquid sheet until an oscillatory state was reached. The temperature of the gases on the right side of the liquid sheet decreases monotonically to a value equal to one, whereas that of the gases on the left oscillates in a periodic manner about a zero value. The

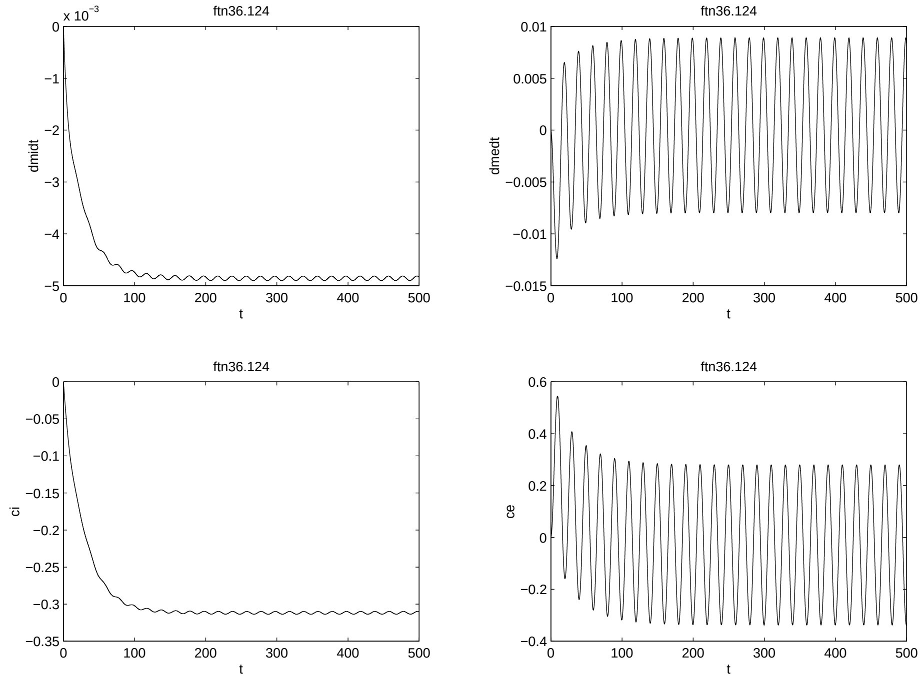

Figure 3: Mass transfer rate for the gases in the right (top left) and left (top right) of the liquid sheet, and interfacial concentration on the right (bottom left) and left (bottom right) sides of the liquid sheet as functions of time. ($Pe_M = 10^6$, $Pe_T = 10^7$, $We = 50$, $Fr = 10$, $\rho_l/\rho_g = 1000$, $C_{pl}/C_{vg} = 100$, $\gamma = 1.4$, $p_i(0)/p_e(0) = 1$, $T_i(0) = 1.5$, $T_e(0) = 1$, $T(0,r,t) = 1$, $c(0,r,t)/(S_i p_i) = 1$, $S_e/S_i = 1$, $S_e/R_g T_{ref} = 1$, $L = a = b = 100$, $\theta(0,t) = 0$).

volume of the gas on the left of the sheet first increases and then slightly decreases and oscillates in a periodic manner, whereas that of the gas on the right first decreases and then oscillates in a periodic manner. For the times presented in Figures 2 and 3, it can be stated that, except for the temperature oscillations, the gases and the liquid have reached the same temperature and that no substantial mass transfer has occurred. Mass transfer is a rather slow process due to the small binary diffusion coefficient of gases in liquids. Once the gases and liquids achieve the same temperature, it may take about one–hundred thousand nondimensional time units to achieve an equilibrium condition characterized by zero pressure differences across the liquid sheet and zero mass transfer.

The flapping motion presented in Figure 2 that indicates that the liquid sheet moves from the centerline to the vertical wall are similar to the oscillatory motions induced when either p_i or p_e are sinusoidal functions of time and there is no heat and mass transfer. In this case, it has been observed that when $p_i(0)/p_e(0) = 1.25$ and the amplitude and frequency of $p_i(t)$ are, for example, 1 and 0.1, respectively, the pressure difference across the liquid sheet is a sinusoidal function of time, but these pressure oscillations result in very small oscillations of the sheet's horizontal displacement, thickness and axial velocity component at the substrate, i.e, at $x = L$, due to the liquid inertia and velocity increase caused by the gravitational pull.

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