EFFECT OF SOLUTE CONCENTRATION LEVEL ON COUPLED MASS AND HEAT TRANSFER DURING SOLID SPHERE DISSOLUTION IN A UNIFORM FLUID FLOW

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Rates of mass and heat transfer between a spherical particle and uniform fluid flow are important in many chemical process that involve dissolution, sublimation, crystallization and heterogeneous chemical reaction. The typical examples of production process involving solid sphere dissolution in liquid are urea, boric, benzoic and butyl benzoic acid dissolution in water, dissolution of uranium spheres in molten cadmium, copper in acid and iron by H_2SO_4. The main goal of this investigation is to obtain simple criterial equations for solid sphere dissolution in uniform fluid flow for Re >>1 which is valid for a case of finite concentration levels of solute in a solvent and for nonisothermal dissolution. The thermodynamic parameters of a system are assumed constant, and only the resistance to mass and heat transfer in the liquid phase is taken into account. Development of thin concentration and temperature boundary layers in liquid starts from the leading edge of a sphere. Convective mass and heat transfer is determined by fluid velocity in the vicinity of solid surface. Heat released or absorbed during solids dissolution is assumed to dissipate in a liquid phase, and it causes change of the liquid temperature. To account for the effects of finite concentration levels of the solute (dissolving solid) in a solvent in dissolution, approach, suggested in papers[1-3] on the effect of absorbate concentration level on mass and heat transfer in film and bubbly absorption is modified. Equation of convective diffusion is derived from a differential mass balance for a dissolving solid and solvent in spherical coordinates. Lateral convective term in the equation of mass transfer of the solute is taken into account. The suggested approach is valid for $Pr = \nu/a >> 1$, $Sc = \nu/D >> 1$, $Pe = 2UR/D >> 1$ and $Le = D/a << 1$, where ν - kinematic viscosity of liquid, a - thermal diffusivity of liquid, D - coefficient of molecular diffusion in liquid, R - radius of a sphere, U - velocity of uniform fluid flow. Boundary conditions at the solid-liquid interface are as follows:

$$x_A = dT + b \text{ at } y = 0 \tag{1}$$

$$\lambda \int_0^{\pi} \left.\frac{\partial T}{\partial y}\right|_{y=0} \sin\theta \, d\theta = L\rho D \int_0^{\pi} \left.\frac{1}{1 - x_{A_s}} \frac{\partial x_A}{\partial y}\right|_{y=0} \sin\theta \, d\theta \text{ at } y = 0 \tag{2}$$

where x_A - weight fraction of dissolved component, x_{A_s} - weight fraction of component A at solid-liquid interface, T - temperature of liquid, L - heat of dissolution, λ - thermal conductivity of liquid, ρ - bulk liquid density, d and b - constants, θ - angular coordinate, y - distance from the surface of a

solid sphere. Finally, dimensionless mass transfer coefficient for nonisothermal dissolution with finite concentration level of solute in a solvent is determined as follows:

$$Sh = \frac{Sh^{is0}}{\gamma^{-1} - \frac{Le^{2/3}}{K}} \tag{3}$$

where

$$\gamma = \frac{Sh^{is}}{Sh^{is0}} = \frac{\Gamma(4/3)}{f(\phi)(1 - x_{A_s})} \tag{4}$$

$$f(\phi) = \int_0^\infty \exp\left(-\left[\alpha^3 - \phi\alpha\right]\right) d\alpha \tag{5}$$

$$\phi = -\frac{1}{1 - x_{A_s}} \cdot \left(\frac{dx_A}{d\eta_c}\right)_{\eta_c = 0} = \frac{x_{A_s} - x_{A_\infty}}{(1 - x_{A_s}) f(\phi)} \tag{6}$$

and Sh^{is0} - Sherwood number for isothermal dissolution with infinite dissolution of solute in a solvent, Sh^{is} - Sherwood number for isothermal dissolution with finite concentration level of solute in a solvent, γ - intensification factor for isothermal dissolution, $K = c_p / dL$, ^{c_p}- specific heat of liquid, ϕ - dimensionless mass average velocity, x_{A_∞}-weight fraction of dissolved component in the bulk of liquid, η_c - similarity variable. $Sh^{is0} = 0.84 Re^{1/2} Sc^{1/3}$ for Re >>1 (see[4]), $Re = 2RU/\nu$ - sphere Reynolds number. Note that L < 0, d > 0 and K < 0 for urea, benzoic acid, boric acid, potassium nitrate, sodium chloride and sodium nitrate dissolution in water. L > 0, d > 0 and K > 0 for zink sulfate, nickel chloride, nickel sulfate, ferric chloride, magnesium chloride, magnesium bromide, magnesium sulfate dissolution in water.

Theoretical results (Eq. (3)-(6)) are consistent with experiments[5] for urea particles dissolution in water in the range of Re number from 50 to 110 (see Fig.1) Authors[5] paid attention that Sherwood number for urea particles dissolution in water is higher than for substances with a low solubility. It is found that the correction factor γ which accounts for the effect of finite concentration level of the solute for a case of isothermal dissolution depends only on solid solubility and initial concentration of the solute in a solvent and is independent on Shmidt number. The correction factor γ represents the enhancement of the rate of mass transfer due to the effect of the additional convective term in equation of convective diffusion. It is shown that for positive K thermal effects lead to the increasement of mass transfer rate in comparison with the isothermal case. On the contrary, for negative K thermal effects lead to the decrease of mass transfer rate in comparison with the isothermal case. The latter effects become more pronounced with the increase of the concentration level of the solute in a solvent. For urea dissolution in water model of isothermal dissolution overestimates the rate of mass transfer on 1.6

% in comparison with nonisothermal model for solids dissolution in liquid with a finite concentration level of solute in a solvent. Criterion of the applicability of applied approach for effects of finite concentration level of solute in a solvent during solids dissolution is as follows:

$$(Sc)^{-1/3}\phi << 1 \qquad (7)$$

It follows from the above analysis that the use of the empirical correlations or models obtained for infinite dilution of solute in a solvent can lead to the essential underestimation of the transfer rates during solids dissolution with a finite solubility, e.g., of the order of tens percents. For example, neglecting the effect of finite concentration level of solute in solvent can lead to the significant errors in estimating the diffusivity of solids in liquids by measuring only initial radius and the time required for the complete dissolution of a solid particle in a case of solids with a finite solubility.

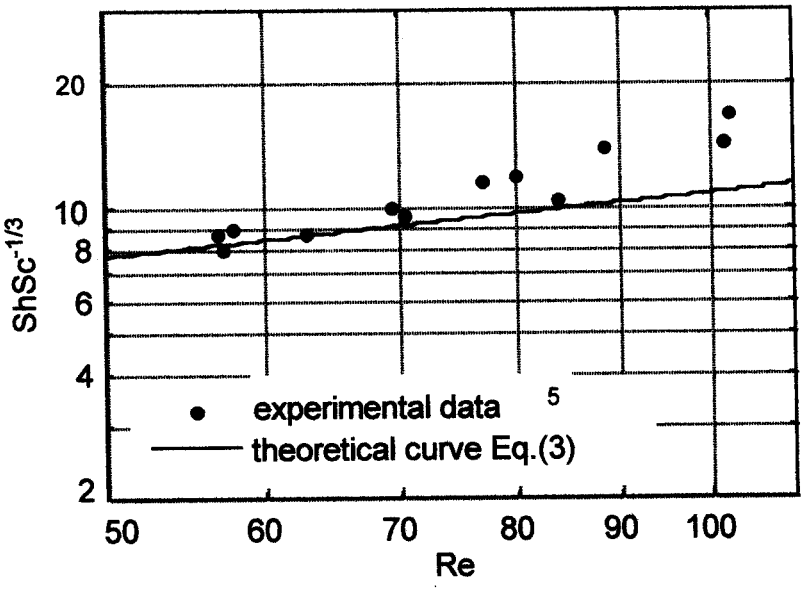

Fig. 1

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