THE ORIGIN OF MULTIPLE SOLUTIONS IN UNSTEADY FREE CONVECTION FLOWS IN POROUS MEDIA

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A great deal of effort has been devoted during the past decades to understanding the convective heat transfer in a fluid-saturated porous medium subject to various additional effects depending on the applications in a variety of geophysical and technological problems. An exhaustive review of the topic of convective flow in porous media can be found in the books by Nield and Bejan (1999), Ingham and Pop (1998, 2002), Vafai (2000) and Pop and Ingham (2001).

Similarity solutions of the boundary layer equations in fluid mechanics and heat transfer theory have proved to be very useful in the interpretation of certain flow and heat transfer characteristics. Although, exact solutions represent highly specialized situations, they do give clues concerning the nature of more realistic behaviour. Here we present new analytic solutions for two of the similarity cases identified by Johnson and Cheng (1978) for the unsteady free convection boundary layer flow over an impermeable vertical flat plate adjacent to a fluid saturated porous medium. These are the asymptotic solutions corresponding to an exponential $\left(e^{a^2 t}\right)$ and a power law $\left(t^m\right)$ variation of the surface temperature respectively, solutions which are valid at sufficiently large distances from the leading edge of the plate. It is shown that in the power law case, physical solutions only exist in the range $m > -1$ of the temperature exponent and they can be expressed in terms of Kummer's confluent hypergeometric functions. For $m \geq 0$ exponentially decaying unique solutions were found, while in the range $-1 < m < 0$ both exponentially and algebraically decaying multiple solutions occur. The origin of the multiple solutions as well as the feasibility conditions of all the solutions mentioned above is discussed in detail.

Key words: unsteady free convection, vertical plate, porous medium, exact similarity solutions, multiple solutions, algebraic decay

References

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