Scattering of light by concave-hull-transformed Gaussian particles

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Abstract

We study light scattering by angular and faceted random particles using the discrete-dipole and geometric-optics methods. For describing the particle shapes, we introduce a concave-hull transformation and apply it to the Gaussian-random-sphere geometry. We describe other potential applications for the concave-hull transformation.

1 Introduction

Naturally occurring small particles provide a world of varying shapes. The Gaussian-random-sphere geometry can be utilized in the modeling of irregular shapes [1]. Based on the Gaussian geometry, first steps are here taken toward modeling angular and faceted shapes. Knowing the scattering characteristics of angular and faceted particles is important, e.g., in solar-system remote sensing.

For modeling the angular and faceted shapes, we define a concave hull: For an arbitrary three-dimensional object, the concave hull coincides with the inner surface formed by a sphere rolling over the object. The concave hull varies as a function of a single scale parameter, the radius of the generating sphere. In the limits of infinitesimal and infinite radii, the concave hull approaches the original shape and the convex hull of the object, respectively. In defining the concave hull, we thus mimic a mechanical profilometer sensing the shape of the object.

We parameterize the concave-hull transformation by the ratio of the generating-sphere radius to a typical radius of the object. For the Gaussian geometry, we utilize the ratio of the generating-sphere radius to the ensemble mean radius, denoting that ratio by h. It follows that the relative curvature radii of the concave-hull concavities cannot be smaller than h. The present concave hull is related to the internal tangenting spheres introduced in [2] to represent the volumes of sample Gaussian particles. For an illustration of the concave-hull transformation, see Fig. 1 for shapes generated with $h = 0$, $h = 2$, and $h = 2 \cdot 10^4$.

In what follows, we show the first application of the concave-hull transformation in discrete-dipole and geometric-optics light-scattering computations.

2 Scattering by concave-hull-transformed Gaussian particles

We parameterize the Gaussian-random-sphere particle with two statistical parameters, the radial-distance standard deviation σ and covariance-function power-law index ν [2]. We assess two pairs of these parameters: first, $\sigma = 0.2$ and $\nu = 2$; and, second, and $\sigma = 0.3$ and $\nu = 4$.

For both geometric-optics and discrete-dipole computations, the complex refractive index of the particles is fixed at $m = 1.55 + i10^{-4}$. For the geometric-optics computations, the size parameter is $x = ka = 100$, where k is the wave number and a is the mean radius of the Gaussian-random-sphere particles. For the discrete-dipole computations concerning individual sample particles in Fig. 1 in random orientation, the equal-volume-sphere size parameter is $x_{\rm ev} = 5$.

The geometric optics computations are carried out for 700 sample shapes with 700 rays incident on each sample particle in random orientation, totaling altogether 490,000 rays. For $\sigma = 0.2$ and $\nu = 2$, decreasing

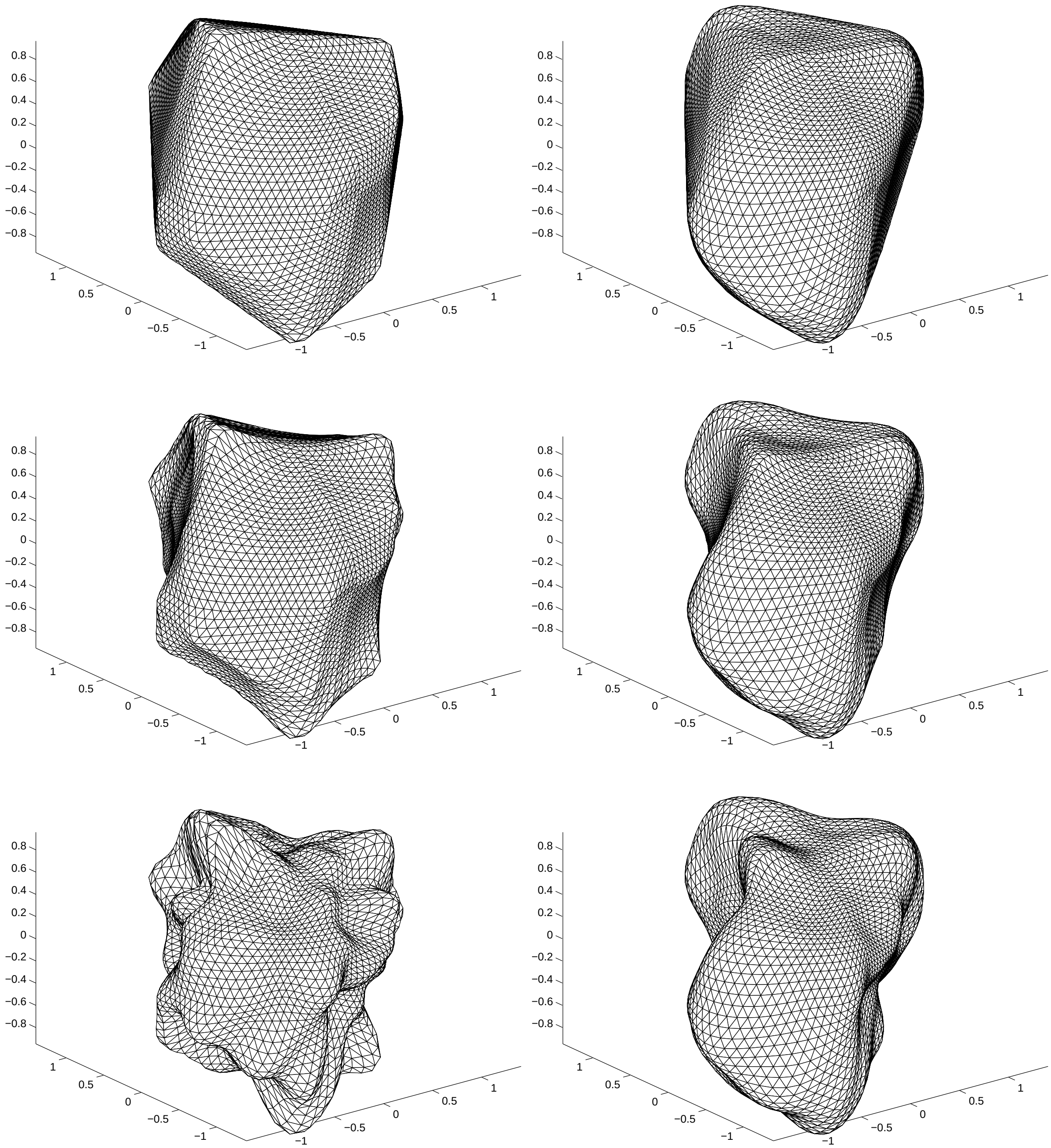

Figure 1: Example shapes corresponding to the Gaussian-random-sphere radial-distance standard deviations and covariance-function power-law indices $\sigma = 0.2$ and $\nu = 2$ (left) as well as $\sigma = 0.3$ and $\nu = 4$ (right). We show the original shapes (bottom; $h = 0$) and their concave hulls generated with spheres of scale radii $h = 2$ (middle) and $h = 2 \cdot 10^4$ (top).

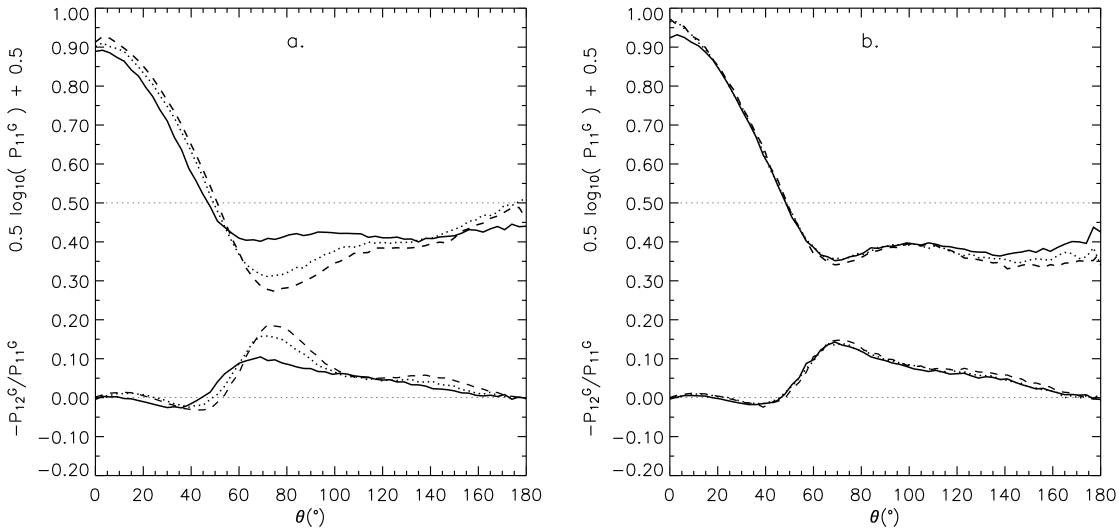

Figure 2: Ensemble-averaged geometric-optics scattering-phase-matrix elements P_{11}^{G} (top) and $-P_{12}^{\mathrm{G}}/P_{11}^{\mathrm{G}}$ (bottom) for random particles originating with (a) $\sigma = 0.2$ and $\nu = 2$ as well as (b) $\sigma = 0.3$ and $\nu = 4$ for the size parameter $x = 100$ and refractive index $m = 1.55 + i10^{-4}$. The solid, dotted, and dashed lines refer to the Gaussian particles and their concave-hull-transformed counterparts with $h = 2$ and $h = 2 \cdot 10^4$, respectively.

concaveness results in increasing steepness of the geometric-optics scattering phase function toward the backscattering direction and increasing degree of linear polarization in the intermediate scattering angles (Fig. 2a). Clearly, the rougher interface more efficiently neutralizes the polarization characteristics. For $\sigma = 0.3$ and $\nu = 4$, the scattering characteristics are perhaps surprisingly independent of the concavities (Fig. 2b).

The discrete-dipole computations are carried out for the shapes depicted in Fig. 1 in 1008 orientations (mimicking random orientation) using the DDSCAT code [3]. For $\sigma = 0.2$ and $\nu = 2$, the discrete-dipole computations show a trend that is the opposite to that in the geometric-optics computations: decreasing concaveness results in decreasing degree of linear polarization for unpolarized incident light in the intermediate scattering angles (Fig. 3a). It is plausible that the original surface with smaller-scale irregularities results in a more pronounced positive polarization due to the electric dipole moments induced in the irregularities. Note the backscattering peaks and negative polarization branches close to backscattering (Figs. 3a and 3b; for explanation, see [2, 4]).

3 Discussion

There are a number of potential applications for the concave-hull geometry presently introduced. The concave hull is unambiguously defined for particles that are aggregates of constituent smaller grains although, in practice, it can become challenging to compute their concave hulls. For rough solid surfaces, for which the convex hull has little significance, the concave hull can be highly useful. For rough particulate surfaces, the concave hull allows studies of porosity as a function of surface height and the concave-hull scale parameter. Finally, for macroscopic objects such as asteroids, the concave hull can help in determining the scale-dependent density of the object.

We have studied light scattering by concave-hull-transformed Gaussian-random-sphere particles showing angular and faceted geometries with the help of the discrete-dipole and geometric-optics methods. The

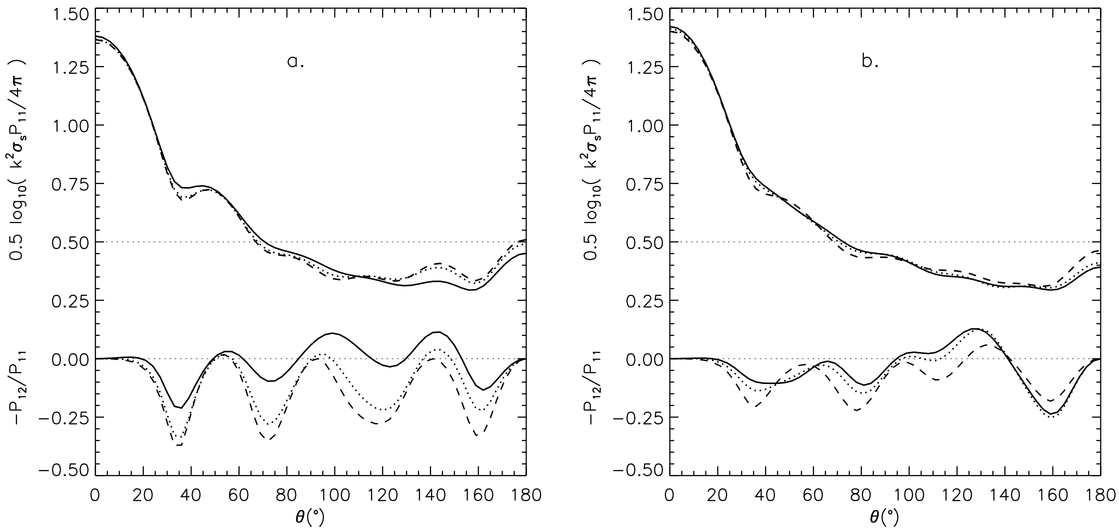

Figure 3: Discrete-dipole scattering-phase-matrix elements $k^2\sigma_s P_{11}/(4\pi)$ (top; σ_s is the scattering cross section) and $-P_{12}/P_{11}$ (bottom) for the particles left and right in Fig. 1 for the equal-volume size parameter $x_{ev} = 5$ and refractive index $m = 1.55 + i10^{-4}$. The solid, dotted, and dashed lines refer to the bottom, middle, and top particle geometries in Fig. 1, respectively.

concave-hull geometry offers a promising tool to study the effects of concavities on the scattering characteristics of nonspherical particles.

Acknowledgments

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References

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