Light scattering properties of a spheroid particle illuminated by an arbitrarily shaped beam

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Abstract

The theories to predict the light scattering properties, such as the scattering diagram, the rainbow angles, the absorption, scattering and extinction cross sections, as well as the radiation pressure forces of a spheroid illuminated by an arbitrarily located, oriented and shaped beam are presented and numerical results are given for different parameters of the particle and the beam.

1 Introduction

The particles encountered in many practical processes, such as the droplets in spray atomization and biological cells, are not spherical. Their optical properties are more or less different from those of spherical ones. The model of spheroid particle is a first order approximation of those non-spherical particles in order to improve the relevant measurement techniques, to better characterize the real particles and to understand the formation of them. When the particle is illuminated by a focused laser beam as in optical tweezers and its size is larger or at the order of radius of the incident beam the non-homogeneous illumination must be considered.

Based on the rigorous solution of Maxwell equations[1] Asano has studied the scattering properties of a spheroid particle when it is illuminated by a plane wave[2]. Barton[3] and Xu *et al*[4-5] have treated the scattering of a laser beam by a spheroid particle in the framework of generalized Lorenz-Mie theory (GLMT). But the rigorous theories are hardly be applied to large particle because of numerical difficulties. The geometrical optics approximation (GOA) is just appropriate to treat such a problem. Lock has studied the diffraction and specular reflection[6] as well as the transmission and cross-polarization effects[7] for an arbitrarily oriented spheroid in the case of plane wave illumination. Xu *et al*, on the other hand, have extended this approach to the scattering of a spheroid for the on-end incidence of a Gaussian beam which permits to predict the scattering diagram in all direction and the rainbow angle for a spheroid of an arbitrary aspect ratio[8].

In this communication we will present the light scattering properties calculated by the rigorous model for small particle and by geometrical optics approximation for large one.

2 Generalized Lorenz-Mie Theory for arbitrarily oriented shaped beam

In General Lorenz-Mie Theory (GLMT), an arbitrarily oriented, located and shaped beam is expanded in spheroidal vector wave functions $\mathbf{M}_{mn}^{(i)}$ and $\mathbf{N}_{mn}^{(i)}$ with two sets of beam shape coefficients $G_{n,TM}^{m}$ and $G_{n,TE}^{m}$. Then all scattering properties can be expressed by the scattering coefficients of the particle A_n^p and B_n^p. For example, the radiation pressure forces are given by:

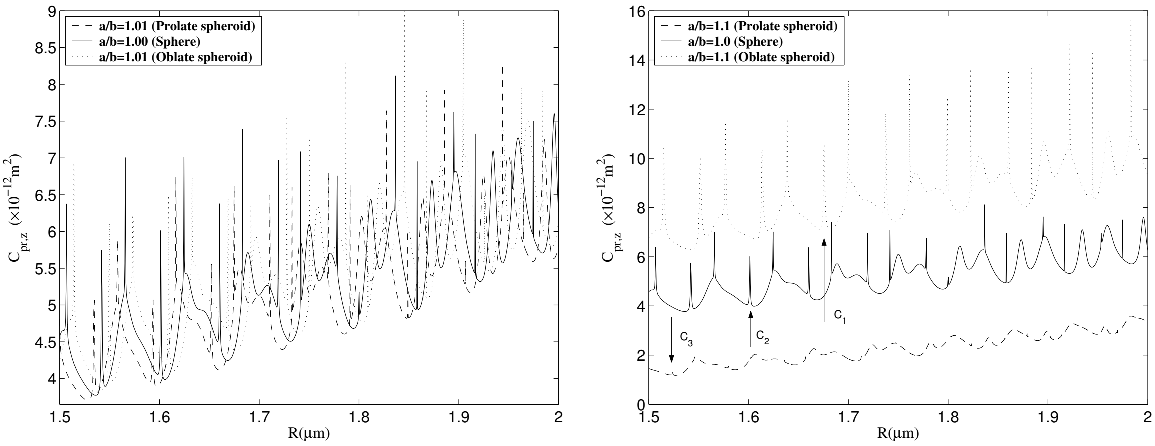
$$C_{pr,x} = \frac{\lambda^2}{4\pi}\sum_{p=1}^{+\infty}\sum_{n=p-1\neq 0}^{+\infty}\sum_{n'=p}^{+\infty}\mathrm{Re}\Big[L_{nn'}^{p-1}\left(2U_{nn'}^{p-1}-S_{nn'}^{p-1}\right)+L_{n'n}^{-p}\left(2U_{n'n}^{-p}-S_{n'n}^{-p}\right)+iM_{nn'}^{p-1}\left(2V_{nn'}^{p-1}-T_{nn'}^{p-1}\right)+iM_{n'n}^{-p}\left(2V_{n'n}^{-p}-T_{n'n}^{-p}\right)\Big],$$

$$C_{pr,y} = \frac{\lambda^2}{4\pi}\sum_{p=1}^{+\infty}\sum_{n=p-1\neq 0}^{+\infty}\sum_{n'=p}^{+\infty}\mathrm{Im}\Big[L_{nn'}^{p-1}\left(2U_{nn'}^{p-1}-S_{nn'}^{p-1}\right)+L_{n'n}^{-p}\left(2U_{n'n}^{-p}-S_{n'n}^{-p}\right)+iM_{nn'}^{p-1}\left(2V_{nn'}^{p-1}-T_{nn'}^{p-1}\right)+iM_{n'n}^{-p}\left(2V_{n'n}^{-p}-T_{n'n}^{-p}\right)\Big],$$

$$C_{pr,z} = \frac{\lambda^2}{4\pi}\sum_{p=-\infty}^{+\infty}\sum_{n=|p|\neq 0}^{+\infty}\sum_{n'=|p|\neq 0}^{+\infty}\mathrm{Re}\Big[J_{nn'}^{p}\left(O_{nn'}^{p}+P_{nn'}^{p}\right)+ipK_{nn'}^{p}\left(Q_{nn'}^{p}-R_{nn'}^{p}\right)\Big],$$

where the coefficients $J_{nn'}^p \sim V_{nn'}^p$ are all expressed in the beam shape coefficients $G_{n,TM}^m$ and $G_{n,TE}^m$ and the scattering coefficients of the particle A_n^p and B_n^p.

Fig. 1 presents the radiation pressure force $C_{pr,z}$ predicted for a sphere, a prolate and an oblate spheroid. The incidence and polarization angles are assumed to be 0˚. The relative refractive index of the particle $\hat{m}$ =1.5. The dashed line is for a prolate spheroid of the semi-minor axis equal to the radius of the projection area (*b*=*R*) and the dotted line is for an oblate spheroid of the semi-major axis equal to the radius of the projection area (*a*=*R*). We find that the oscillation of the prolate spheroid is much weaker for laser beam illumination than for plane wave case.



(a) plane wave incidence (b) beam incidence case

Figure 1. Longitudinal radiation pressure cross section $C_{pr,z}$ exerted by the plane wave on the spheroid. For clarity and convenient identification, the RPCS curves of the prolate and oblate spheroids have been offset by the factors 2×10^{-12}, -2×10^{-12} respectively.

On the other hand, according to the discontinuity of the electromagnetic field, the surface force can also be calculated by the GLMT. We can then predict the deformation of a biological cell in the optical tweezers for example.

3 Extension of geometrical optics approximation

In the extension geometrical optics approximation (EGOA)[8], a shaped beam is considered as a bundle of rays. Each of them is reflected or refracted at the surface of the particle. The scattered wave is then the sum of all the rays in considering the amplitude and the phase of each ray.

The scattering intensity I_j of j^{th} ray in the far-field at an observation point of distance r from the particle center is given by:

$$I_j = \frac{I_0}{(kr)^2}i_j(\theta) = \frac{I_0}{(kr)^2}\left|S_j(\theta)\right|^2$$

where the complex amplitude $S_j = S_d + \sum_{p=0}^{\infty} S_{j,p}$, S_d is the diffracted contribution, S_j is the amplitude of the ray of order *p*.

It should be pointed out that when the particle is illuminated by a shaped beam, the phase in the complex amplitude is composed of three parts: the phase shift due to the optical path $\phi_{p,PH}$, the phase shift due to the focal lines (inside and out of the sphere) $\phi_{p,FL}$ and that due to the wave front curvature of the shaped beam ϕ_G :

$$\phi_p = \left(\frac{\pi}{2}\right) + \phi_{p,PH} + \phi_{p,FL} + \phi_G$$

A scattering diagram of a sphere illuminated by a Gaussian beam calculated by GLMT and EGOA for the on-axis case. We find that when the beam radius is smaller than the particle radius the agreement of the two methods is very good. But when the beam is much larger than the particle radius the discrepancy becomes remarkable around the rainbow angle and 90°.

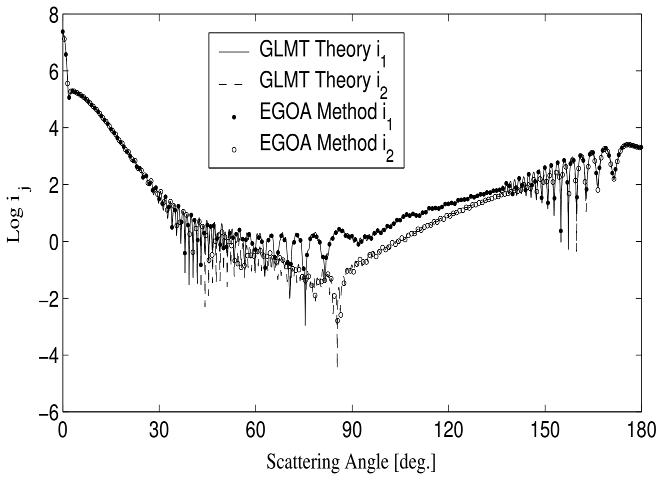


Figure 2. Comparison of the scattering intensities calculated by GLMT and EGOA for a pure water droplet of refractive index $m = 1.33$ and radius $a = 25$ μm illuminated by a Gaussian beam of waist radius $w_0 = 10$ μm and wavelength $\lambda = 0.6328$ μm. The particle is located at the center of the beam.

It is well known that the rainbow angle is very sensitive to the form of the particle. Möbius has proposed a model to predict the rainbow position of a spheroid of aspect ratio near unity. With the EGOA we can predict the rainbow angle for any aspect ratio.

The primary rainbow position of a spheroid of water droplet (m=1.33) with on-end incidence of a Gaussian beam predicted by EGOA is shown in Fig. 3. We can distinct three zones. When the aspect ratio is less than 1.4 the rainbow angle increases as function of the aspect ratio and the formation mechanism is the same as for a spherical particle. There is no primary rainbow when the aspect ratio is between 1.4 and 1.65. The rainbow reappears for the spheroid of aspect ratio larger than 1.65. The rainbow angle decreases as function of the aspect ratio and the formation mechanism is not the same. We find also that the rainbow position is not sensitive to the beam radius.

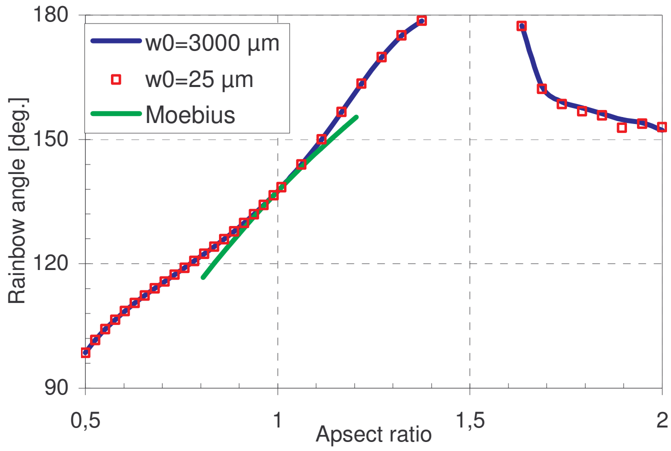


Figure 3. Primary order rainbow position versus aspect ratio, predicted by EGOA method for a spheroid droplet of projection radius R=100µm and illuminated by the Gaussian beam of waist radii w_0=25, and 3000µm, respectively.

4 Conclusion

The light scattering properties of a spheroid illuminated by a shaped beam can be predicted by the rigorous theory in the framework of generalized Lorenz-Mie theory for relatively small particle or by geometrical optics approximation for large particle. Some numerical results are exemplified.

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