Backscattering of light from a layer of densely packed random medium

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Abstract

The theory of light scattering by systems of nonspherical particles is applied to derive equations corresponding to incoherent (diffuse) and interference parts of radiation reflected from the medium. A solution of the system of linear equations describing the light scattering by a system of particles is represented by iteration. The symmetry properties of the T-matrices and of the translation coefficients for the vector Helmholtz harmonics lead to the reciprocity relation for an arbitrary iteration. This relation is applied to consider the backscattering enhancement phenomenon. In the exact backscattering direction the relation between incoherent and interference parts is identical to that for sparse media.

The problem of light scattering by densely packed discrete random media is important for many areas of science and technology. In particular this problem is of interest to astrophysics. The surfaces of the majority of atmosphereless solar system bodies are covered with regolith and the interpretation of optical observations of them must be based on an adequate theory of light scattering by a densely packed medium. Some of these bodies exhibit the opposition effect (or the effect of coherent backscattering [1,2]). The effect of coherent backscattering has been observed in numerous laboratory experiments as well [2]. It is caused by the constructive interference of multiple scattered waves traveling in a discrete medium in a certain direct and reversal trajectories [1,2]. The well known Saxon's reciprocity relation [3] is applied to investigate of the effect of coherent backscattering. Although the Saxon's reciprocity relation is valid for arbitrary finite scatterers including systems of particles, it is valid only in the far-field zone of the scatterers. In theoretical analysis of multiple scattering of waves by media this relation can be applied only for sparse media whose particles are located in the far-field zones of the each other. Because of this, the effect of coherent backscattering is comparatively well studied for such media. In particular, for sparse media a relation between the incoherent and coherent parts of scattered light in the exact backscattering direction has been obtained [4,5]. This relation allows one to calculate the amplitude of the interference peak of the coherent backscattering effect using the vector radiative transfer equation only [4]. Equations for describing the angular dependence of the interference part of multiply scattered waves by a layer of sparse medium of randomly oriented arbitrary particles have been obtained in [6,7].

In this work the theory of light scattering by densely packed systems (aggregates) of nonspherical particles is applied to study the light scattering by densely packed media. It is assumed that the $\mathbf{T}$-matrices of all aggregate particles are known in their local coordinate systems and that the smallest circumscribing spheres of all particles do not overlap with each other. A solution of the system of equations describing scattering of waves by particles is represented by a method of iteration. It is shown that the symmetry properties of the $\mathbf{T}$-matrices and of the translation coefficients of the vector Helmholtz harmonics lead to the reciprocity relation for an arbitrary iteration. For the amplitude matrix of an iteration this relation is identical to the Saxon's reciprocity relation. For example, let $S_{pn}^{(j,\sigma,s)}(\mathbf{k}_0,\mathbf{k}_{sc})$ be the amplitude scattering matrix for the wave propagating in the direction $\mathbf{k}_0$ with initial polarization n, scattered first by particle s, then by particle σ and finally by particle j in the direction $\mathbf{k}_{sc}$ with final polarization p. (Here the so-called CP-representation is used in which the polarization indexes $n, p = \pm 1$). In this case the reciprocity relation can be written as

$$S_{pn}^{(j,\sigma,s)}(\mathbf{k}_0, \mathbf{k}_{sc}) = S_{np}^{(s,\sigma,j)}(-\mathbf{k}_{sc}, -\mathbf{k}_0)\,. \tag{1}$$

The symmetry relation for the amplitude scattering matrix (1) is applied to consider the backscattering enhancement phenomenon for macroscopically homogeneous, isotropic and mirror-symmetric medium. Let the medium be in form of plane-parallel layer and the upper boundary of the medium coincides with the plane ($x_0 y_0$) of the coordinate system $\hat{\mathbf{n}}_0$ (Fig.1). In the paper bold letters with carets $\hat{\mathbf{n}}_i$ denote right-handed coordinate systems (x_i, y_i, z_i) with the z_i-axes along the vectors $\mathbf{n}_i$. The scattering matrix, which transforms the Stokes parameters of the incident light into those of the scattered light is supposed to be in the form:

$$S_{pn\mu\nu} = S_{pn\mu\nu}^{(L)} + S_{pn\mu\nu}^{(C)}\,. \tag{2}$$

Here matrix $S_{pn\mu\nu}^{(L)}$ describes the diffuse (incoherent) part of scattered light and matrix $S_{pn\mu\nu}^{(C)}$ describes the coherent (interference) part. $p, n, \mu, \nu = \pm 1$.

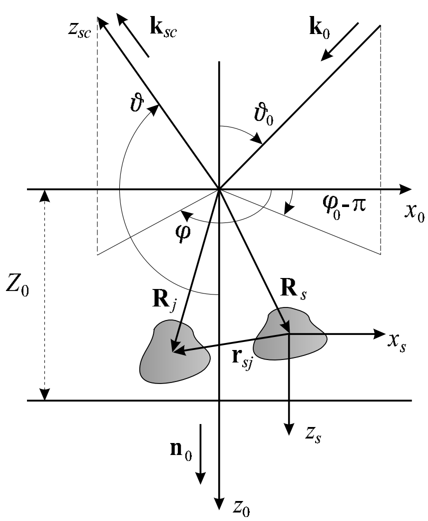

Figure 1. Geometry of scattering of light by a layer.

Equations for matrixes $S_{pn\mu\nu}^{(L)}$ and $S_{pn\mu\nu}^{(C)}$ are obtained using the quasi-crystalline approximation and an approach described in [6,7]. Radii of the circumscribing spheres of all particles are assumed to be identical.

$$S_{pn\mu\nu}^{(L)} = \frac{\eta}{k_0} \sum_{LMlm} \frac{(2L+1)(2l+1)}{4} D_{Mp}^{*L}(\varphi, \vartheta, 0) D_{m\mu}^{l}(\varphi, \vartheta, 0) \int_0^{k_0 Z_0} B_{LMlm}^{(z)(pn)(\mu\nu)} \exp\left(\frac{\tau z}{\cos\vartheta}\right) dz\,, \tag{3}$$

where η is the concentration of particles, $k_0 = 2\pi/\lambda$, $\tau = 2\,\mathrm{Im}(m_{eff})$, $m_{eff} = \mathrm{Re}(m_{eff}) + i\,\mathrm{Im}(m_{eff})$ is the effective refractive index of the medium, Z_0 is the thickness of the medium. Matrix $S_{pn\mu\nu}^{(L)}$ is defined per unit area of the upper boundary of the medium. $D_{Mn}^{L}(\varphi, \vartheta, \gamma)$ is the Wigner function, $\varphi, \vartheta, \gamma$ are the Euler angles determining the rotation from the coordinate system $\hat{\mathbf{n}}_0$ into coordinate system $\hat{\mathbf{k}}_{sc}$ (Fig.1).

Coefficients $B_{LMlm}^{(z)(pn)(\mu\nu)}$ are determined from the system of equations

$$
\begin{aligned}
B_{LML_1M_1}^{(z)(pn)(\mu\nu)} &= \exp\left(-\frac{\tau z}{\cos\vartheta_0}\right)\sum_{lml_1m_1} t_{LMlm}^{(pn)} t_{L_1M_1l_1m_1}^{*(\mu\nu)} D_{mn}^{l}(\varphi_0,\vartheta_0,0) D_{m_1\nu}^{*l_1}(\varphi_0,\vartheta_0,0) + \\
&+\eta \sum_{qq_1lml_1m_1} t_{LMlm}^{(pq)} t_{L_1M_1l_1m_1}^{*(\mu q_1)} \sum_{l_2m_2l_3m_3} \int g(r) B_{l_2m_2l_3m_3}^{(y)(qn)(q_1\nu)} H_{lml_2m_2}^{(q)}(\hat{\mathbf{n}}_0,\hat{\mathbf{r}}) H_{l_1m_1l_3m_3}^{*(q_1)}(\hat{\mathbf{n}}_0,\hat{\mathbf{r}}) d\mathbf{r},
\end{aligned}
\tag{4}
$$

where $y = z + k_0 r\cos\omega$, the angle ω $(0 \le \omega \le \pi)$ is measured from the direction $\mathbf{n}_0$ (see Fig.1), $q, q_1 = \pm 1$, $g(r)$ is the pair distribution function, $H_{lml_2m_2}^{(q)}(\hat{\mathbf{n}}_0,\hat{\mathbf{r}})$ are the translation coefficients for the vector Helmholtz harmonics (see, for example, [6]). Integration in Eq.4 is over the whole volume of the medium. $t_{LMlm}^{(pn)} t_{L_1M_1l_1m_1}^{*(\mu\nu)} = \left\langle t_{LMlm}^{(j)(pn)} t_{L_1M_1l_1m_1}^{*(j)(\mu\nu)} \right\rangle$ where the angular brackets denote averaging over particle orientation and particle properties,

$$
t_{LMlm}^{(j)(pn)} = -i^{l-L}\sqrt{\frac{2l+1}{2L+1}}\left[T_{MLml}^{11(j)} + nT_{MLml}^{12(j)} + pT_{MLml}^{21(j)} + pnT_{MLml}^{22(j)}\right],
\tag{5}
$$

and $T_{MLml}^{kw(j)}$ $(k,w=1,2)$ are the elements of the $\mathbf{T}$-matrix of j-th particle. For sparse media Eqs.(3),(4) can be transformed into the classical vector radiative transfer equation [4,6,7], and in the case of normal illumination of semi-infinite medium of densely packed identical spherical particles they can be transformed into the equations obtained in Ref.8.

$$
S_{pn\nu\mu}^{(C)} + S_{pn\nu\mu}^{(1)} = \frac{\eta}{k_0}\sum_{LMlm}\frac{(2L+1)(2l+1)}{4} D_{Mp}^{*L}(\varphi,\vartheta,0) D_{m\mu}^{l}(\pi+\varphi_0,\pi-\vartheta_0,0)\int_0^{k_0Z_0} F_{LMlm}^{(z)(pn)(\mu\nu)}\exp(-i\varepsilon z)\,dz,
\tag{6}
$$

where

$$
\varepsilon = \frac{m_{eff}-1}{\cos\vartheta} + \frac{m_{eff}^{*}-1}{\cos\vartheta_0},
\tag{7}
$$

and coefficients $F_{LMlm}^{(z)(pn)(\mu\nu)}$ are determined from the system of equations

$$
\begin{aligned}
F_{LML_1M_1}^{(z)(pn)(\mu\nu)} &= \exp\left(i\varepsilon^{*} z\right)\sum_{lml_1m_1} t_{LMlm}^{(pn)} t_{L_1M_1l_1m_1}^{*(\mu\nu)} D_{mn}^{l}(\varphi_0,\vartheta_0,0) D_{m_1\nu}^{*l_1}(\pi+\varphi,\pi-\vartheta,0) + \\
&+\eta \sum_{qq_1lml_1m_1} t_{LMlm}^{(pq)} t_{L_1M_1l_1m_1}^{*(\mu q_1)} \sum_{l_2m_2l_3m_3} \int g(r) F_{l_2m_2l_3m_3}^{(y)(qn)(q_1\nu)} H_{lml_2m_2}^{(q)}(\hat{\mathbf{n}}_0,\hat{\mathbf{r}}) H_{l_1m_1l_3m_3}^{*(q_1)}(\hat{\mathbf{n}}_0,\hat{\mathbf{r}}) \exp\left(i\mathbf{r}(\mathbf{k}_0+\mathbf{k}_{sc})\right) d\mathbf{r}.
\end{aligned}
\tag{8}
$$

Matrix $S_{pn\nu\mu}^{(1)}$ in Eq.(6) corresponds to the single scattering (the first term on the r.h.s. of Eq. (8)). For sparse media Eqs.(6),(8) can be transformed into the equations obtained in Refs.6,7.

In the case of $\mathbf{k}_{sc} = -\mathbf{k}_0$ the system (8) is identical to the system (4). Therefore the following equation is valid

$$
S_{pn\mu\nu}^{(L)} = S_{pn\nu\mu}^{(C)} + S_{pn\nu\mu}^{(1)}.
\tag{9}
$$

This equation exactly reproduces Eq.(19) in Ref.9 for sparse media. In the case of $\mathbf{k}_{sc} \ne -\mathbf{k}_0$ the multiplier $\exp\left(i\mathbf{r}_{js}(\mathbf{k}_0+\mathbf{k}_{sc})\right)$ in Eq.(8) rapidly oscillate and the contribution of the second term of this equation tends to zero. As result the matrix $S_{pn\nu\mu}^{(C)}$ in Eq. (6) differs from zero only in the narrow area of scattering angles near the direction $\mathbf{k}_{sc} = -\mathbf{k}_0$. In intensity of scattered light this matrix describes a narrow interference peak centered in the exact backscattering direction.

Eqs.(3)-(8) describe the incoherent (diffuse) and coherent parts of reflected radiation from a plane parallel layer of densely packed particles. For sparse media these parts determine the reflection matrix of light. For such media the incoherent part corresponds to the sum of the ladder diagrams and is described by the vector radiative transfer equation [4]. The interference part of scattered radiation corresponds to the sum of the cyclical diagrams and is described by equations obtained in [6,7]. In the intensity of scattered light, this part appears as a narrow peak centered at the exact backscattering direction. In the exact backscattering direction the interference part is related with incoherent part by a simple equation [4,5,9]. The same equation (Eq.(9)) is valid for densely packed media as well. However, for densely packed media, the equations describing the incoherent and coherent parts are much more complex. Complexity of the equations is caused by the correlation between particles and by the inhomogeneity of waves near the particles. The inhomogeneity of waves near the particles is described by the coefficients of the addition theorem for the vector Helmholtz harmonics. For sparse media these coefficients have a simple form and describe the propagation of spherical waves between particles [6,7]. For densely packed media of particles comparable in sizes to the wavelength, the equations for incoherent and coherent parts describe only a part of scattered radiation. Additional part of scattered radiation for such media can come, for example, from the interference of waves scattered by neighboring particles and interference of waves of different orders of scattering [8]. Unfortunately, consideration of such interferences is a very complicated problem and is far from being satisfactory resolved even for qualitative analysis.

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