Comparison of LS Methods Using Single Expansions of Fields

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Abstract

The light scattering methods expanding the fields in terms of wave functions are widely applied due to their high efficiency. We compare some of these methods, namely the extended boundary condition, separation of variables, and point matching ones, considering their theoretical and practical applicability. Though the methods look alike because of their use of the same expansions of the fields, it is found that these approaches differ in important aspects.

1 Introduction

The approaches using single expansions of the fields in terms of wave functions to solve the light scattering problem are the extended boundary condition (EBCM), separation of variables (SVM), and point matching (PMM) methods [1, 2]. They give solutions widely used because of their high speed and accuracy.

These approaches seem to be very similar as they search for the problem solution in the form of the *same* field expansions with the expansion coefficients being derived from solution of the systems of linear algebraic equations. The main difference of the methods is their use of different problem formulation which leads as a result to different systems. In the EBCM the fields and Green function expansions are substituted into the boundary condition presented in the surface integral form; in the SVM the field expansions are substituted into the boundary condition written in the usual (differential) form; in the PMM any of the above forms can be used, but the system is derived from minimization of a residual of the boundary condition considered in selected points at the scatterer surface (see [2] for more details).

In this work we compare the methods using the spherical function basis by considering their theoretical and practical applicability ranges. In the latter case the questions of accuracy and computational time are concerned. Note that though the EBCM, SVM, and PMM represent the fields in the same way, a different number of terms in the expansions may be required by the methods to reach the same accuracy.

2 Theoretical aspects

From the theoretical point of view, applicability of the approaches is determined by *convergence* of the field expansions and *solvability* of the systems used to derive the expansion coefficients.

The convergence of the expansions of the scattered and internal fields at a distance d from the coordinate origin does not depend on the methods used and occurs if and only if

$$\max\{d^{\rm sca}\} \;<\; d \;\; \text{and} \;\; d \;<\; \min\{d^{\rm int}\}, \tag{1}$$

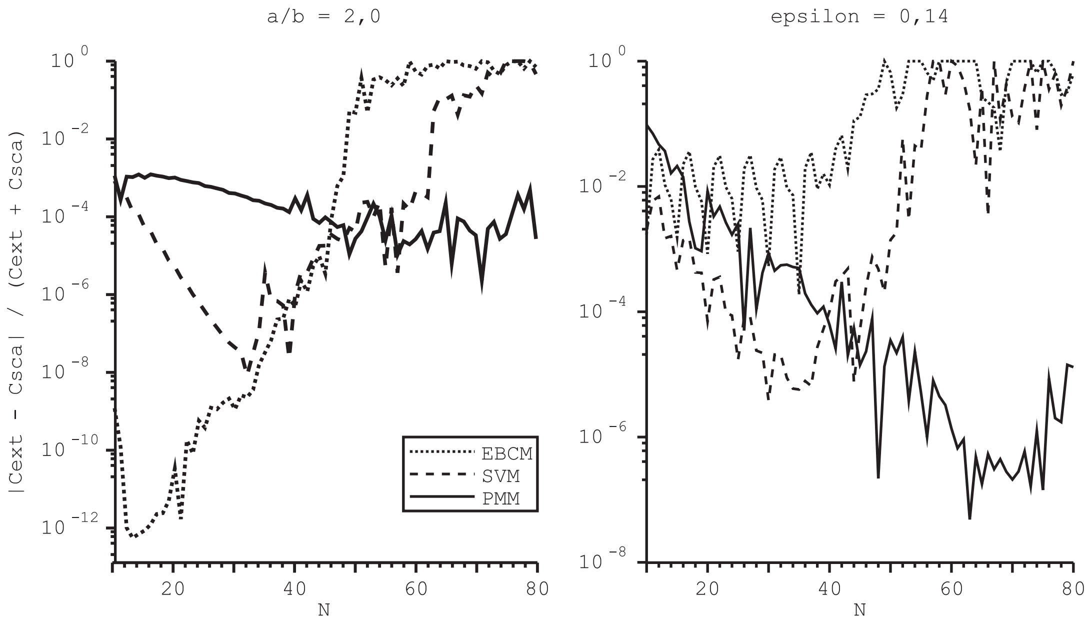

Figure 1: The dependence of the code accuracy measure δ on the number of terms kept in the field expansions N for spheroids with the aspect ratio $a/b = 2$ (left panel) and Chebyshev particles with $n = 5$ and $\varepsilon = 0.14$ (right panel). For both scatterer types, refractive index is $m = 1.5$, the diffraction parameter $x_{\mathrm{v}} = 1$, and the radiation incidence angle $\alpha = 10°$.

where d^{sca} and d^{int} are the distances to singularities of analytic continuations of the fields (see [3] for more details).

The solvability of the system arising in the EBCM takes place provided (see the discussion in [2])

$$\max\{d^{\mathrm{sca}}\} \ < \ \min\{d^{\mathrm{int}}\}. \tag{2}$$

For the SVM, a similar analysis has not yet been performed, though at least the EBCM system can be obtained within the SVM [3]. The system arising in the PMM is always solvable, as it is positively determined (see, e.g., [2]).

It should be added that the convergence is important for calculations of the fields in the near field zone (note that the condition (2) is always weaker than the condition (1)). The solvability plays the main role in the far field zone where the field expansions should converge. For instance, any EBCM calculations of the field characteristics in this zone converge only if the condition (2) is satisfied [2]. Note also that the conditions (1), (2) do not depend on such scatterer characteristics as refractive index and diffraction parameter as only the geometrical parameters are involved.

3 Practical aspects

We have developed a homogeneous set of codes based on the methods under consideration. Their accuracy in the far field zone was studied by using the relative difference of the extinction and scattering cross sections $\delta = |C_{\mathrm{ext}} - C_{\mathrm{sca}}| / (C_{\mathrm{ext}} + C_{\mathrm{sca}})$ calculated for nonabsorbing particles.

The typical behaviour of accuracy of the codes with a changing number of terms kept in the expansions N is shown in Fig. 1. For spheroids, the EBCM solution well converges with growing N and gives as high accuracy as $\sim 10^{-12}$ for $N \approx 16$. For $N \gtrsim 16$, the system is ill-conditioned and the

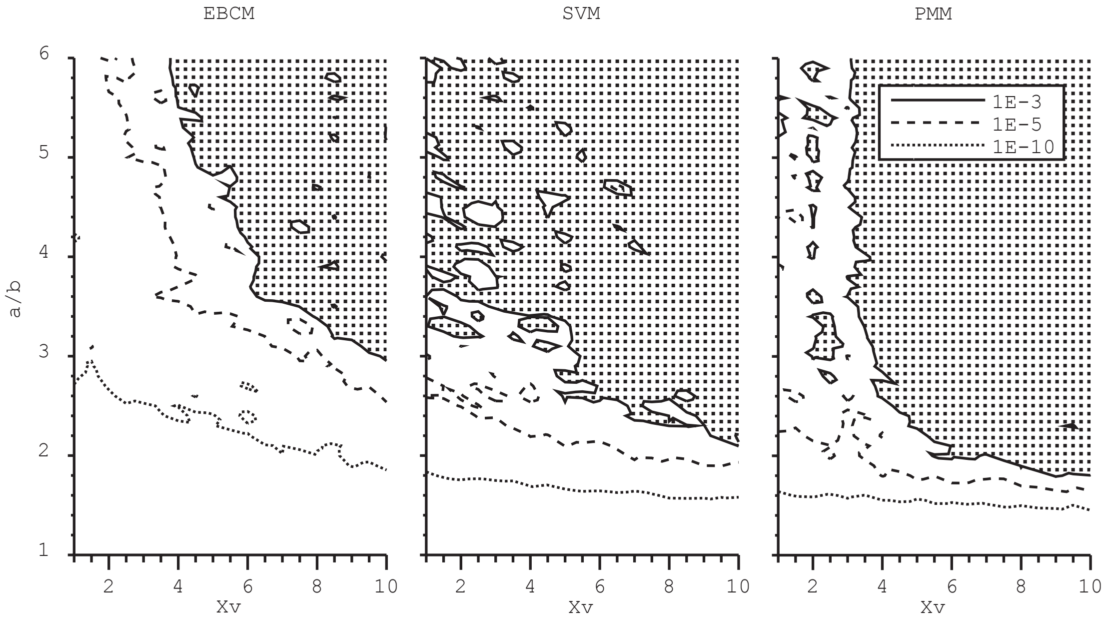

Figure 2: Dependence of δ on prolate spheroid parameters a/b and x_{v} ($\alpha = 10°$, $m = 1.5$).

accuracy quickly drops with increasing N. The same behaviour is observed for the SVM solution, though it reaches maximum accuracy when $N \sim 30$. For the PMM solution, the convergence is even slower and numerical problems appearing at $N \gtrsim 40$ limit its practical applicability.

Note that for any spheroids all 3 methods are mathematically correct [2]. The situation is a bit different for axisymmetric Chebyshev particles having the surface equation $r(\theta) = r_0(1 + \varepsilon \cos n\theta)$, where θ is an angle of the spherical coordinates, r_0 the radius of a unperturbed sphere, ε its deformation amplitude, n the number of surface maxima.

The EBCM solvability condition (2) is satisfied in a certain region of the geometrical parameters of the particles, and in particular for $\varepsilon < 0.14$, when $n = 5$ [2]. Our calculations well confirm this, e.g. Fig. 1 shows that accuracy of the EBCM on average does not grow with increasing $N < 40$. Meantime the SVM and PMM solutions converge for $N < 35$ and $N < 65$, respectively. Thus, despite the large similarity of the EBCM and SVM (see the discussions in [2,3]), their theoretical applicability conditions differ in principle as definitely the solvability of the SVM is not determined by the EBCM condition (2).

A systematic numerical comparison of the methods is still absent in the literature. As a first step of such comparison in Fig. 2 and 3 we consider the regions of the parameters of spheroids and Chebyshev particles where the methods can provide a given accuracy. The figures allow one to see the practical applicability of the methods. For spheroids, the EBCM is generally preferable to other 2 methods. In contrast, for Chebyshev particles, the SVM and PMM look better (the former needs less terms and hence is faster, however the latter can give higher accuracy). Therefore, in a general case it is worth to combine these 3 methods keeping in mind that their codes differ by a few dozen operators.

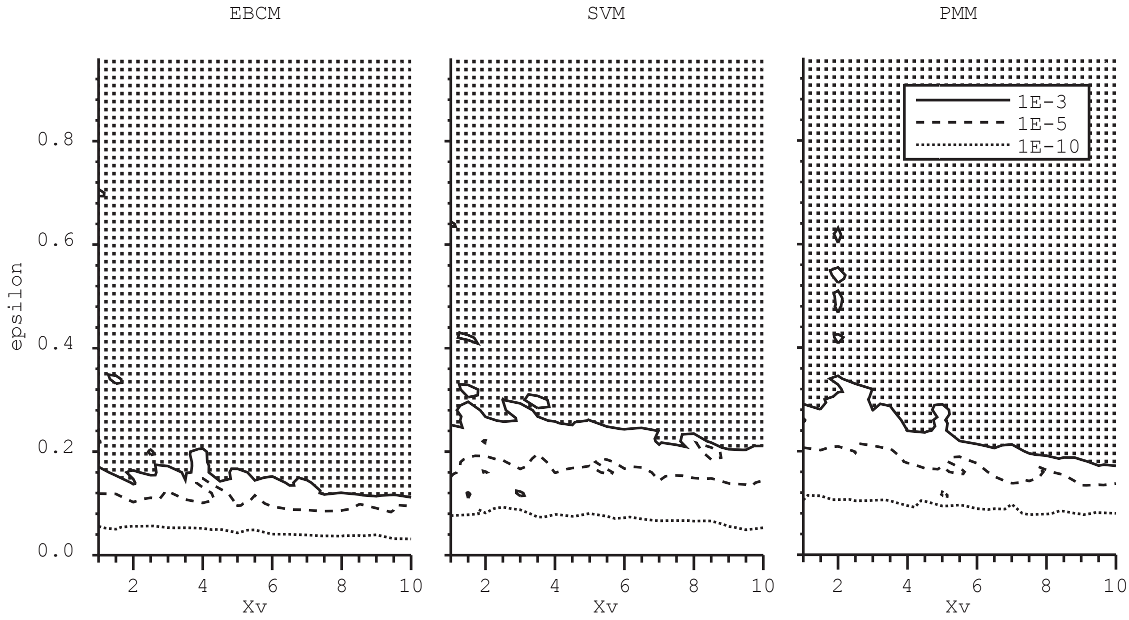

Figure 3: Dependence of δ on Chebyshev particle parameters ε and x_v ($n = 5$, $\alpha = 10^\circ$, $m = 1.5$).

4 Conclusions

We have compared 3 methods utilizing single field expansions in terms of spherical wave functions. It is found that despite a very large similarity of the EBCM and SVM their theoretical applicability ranges are different in principle.

Extensive calculations have shown that the methods well supplement each other, and as the codes differ by a few operators it is worth to combine them.

Our preliminary results of similar comparison of the methods, when the spheroidal functions were used for the field expansions, led to the same conclusions.

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