SPARSE FINITE ELEMENTS FOR NON-SCATTERING RADIATIVE TRANSFER IN DIFFUSE REGIMES

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ABSTRACT. The stationary monochromatic radiative transfer equation is posed in five dimensions, with the intensity depending on both a position in a three-dimensional domain as well as a direction. In order to overcome the high dimensionality of the problem, we propose a new multiscale Galerkin Finite Element discretizaton that, for non-scattering radiative transfer problems in diffuse regimes, reduces the complexity of the problem to the number of degrees of freedom in physical space only (up to logarithmic terms).
Numerical test examples for translation invariant problems with respect to the z-axis show that our method is clearly superior to a full least-squares finite-element discretization: essentially the same accuracy with respect to incident radiation and heat flux is achieved with significantly fewer degrees of freedom.

NOMENCLATURE

I_b	blackbody intensity
n	dimension in physical space (= 2 or 3)
D	domain $\subset \mathbb{R}^n$
∂D	boudary of domain D
$\mathbf{n}$	outer unit normal on the boundary of the domain D
S^2	unit sphere in $\mathbb{R}^3$
$\mathbf{x}$	$(x, y, 0)'$ for $n = 2$, $(x, y, z)'$ for $n = 3$
$\mathbf{s}$	$(\cos\varphi \sin\vartheta, \sin\varphi \sin\vartheta, \cos\vartheta)' \in S^2$
$\boldsymbol{\nabla}_{\mathbf{x}}$	$(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, 0)'$ for $n = 2$, $(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})'$ for $n = 3$
Γ_-	inflow boudary $(\mathbf{s}, \mathbf{x}) \in S^2 \times \partial D$ with $\mathbf{s} \cdot \mathbf{n} < 0$
$\|\|u\|\|$	$:= \sqrt{\int_D \int_{S^2} \|u(\mathbf{x}, \mathbf{s})\|^2 \, d\mathbf{x} \, d\mathbf{s}}$
$\|\|u\|\|_{H^{1,0}}$	$:= \sqrt{\|\|\boldsymbol{\nabla}_{\mathbf{x}} u\|\|^2 + \|\|u\|\|^2}$
$supp(u)$	support of function u
N_D	number of degrees of freedom in physical space
N_{S^2}	number of degrees of freedom in solid angle