THE ANALYSIS OF THE SPECTRAL RADIATIVE HEAT TRANSFER IN OPTHICALLY THICK SEMITRANSPRENT PLANAR MEDIA

Nuray Kayakol

Technische Universität Darmstadt
Energy Systems and Technology
Petersenstraße 30
D-64287 Darmstadt

Abstract
One-dimensional analysis of combined conduction and radiation is carried out in a planar semitransparent medium by using the discrete ordinates method and effective thermal conductivity approach, which is commonly used for the simulation of the glass melting tanks. The band-wise solution of radiative transfer equation is applied. The predictions are compared with effective thermal conductivity approach. The discrete ordinates method over predicts temperature distribution for dark glasses having high optical thickness. It needs to be modified for highly optically thick semitransparent media.

1. INTRODUCTION

The combined conduction and radiation heat transfer in melting tank of a glass manufacturing system is a problem of considerable importance. Glass is semitransparent to thermal radiation in approximately the spectral range of $0 < \lambda < 5$ μm, and is effectively opaque beyond 5μm. The most popular prediction of heat transfer in glass melts is the concept of effective thermal conductivity (KEFF). Its popularity comes from the advantage of mathematical simplicity due to a diffusion formulation for radiation heat transfer. However, it is well known that diffusion approximation fails near the boundaries [1]. Alternatively, a rigorous analysis may be carried out by coupling the solution of energy equation with the solution of radiative transfer equation through a source term which is the divergence of radiative flux. The discrete ordinates method (DOM) is popular for predicting radiative transfer in participating media [2] and applied to semitransparent media [3]. Spectral DOM is recommended especially for transparent glasses which has low absorption coefficient [4]. The method needs to be tested for dark glasses high optical thickness.

Spectral variation of absorption coefficient of a glass medium can be handled by using absorption coefficient versus wavelength data at different temperatures for different glass types having different metal ions like Fe+2, Cr+3 content which gives colour to glass melt. The absorption coefficient varies gradually with wavelength. [5]

Spectral analysis of glass medium is computationally cheap compared to that of combustion space. It doesn't require a radiative property estimation model. Experimental spectral data can be directly applied to the band-wise solution of radiative transfer equation. The spectrum can be divided into a few bands where the absorption coefficient is expressed as constant or simple function of temperature. RTE is solved for each band and total heat transfer is just the summation of the results over all bands. Accuracy depends on number of bands. Three number of band is recommended [6].

The purpose of the present study is to analyse spectral radiative heat transfer in a semitransparent planar medium for the evaluation of DOM. The DOM method is considered to be numerically correct. However it is not tested for very high optical thicknesses. Performance of DOM is tested for optically thick media where absorption coefficient is between than 5-10 cm^{-1}. Three types of commercial glasses, colourless, light green and dark green were chosen for the study. The non-gray character of these glasses is taken into account by dividing their electromagnetic spectrum, ranging between 0.5 μm-4.5 μm, into three distinctive bands. The temperature predictions of DOM are compared with those obtained by KEFF approximation.

2. DESCRIPTION OF THE METHOD

2.1 Mathematical Formulation

The 1-D energy conservation equation for conduction and radiation heat transfer is

$$\frac{d}{dx}\left[-\frac{dT}{dx}+q_r\right]=0 \quad (1)$$

where q_r is the total radiative heat flux, which is obtained by integrating the spectral radiative heat flux distribution over the entire spectrum

The Radiative Transfer Equation, RTE, written for non-scattering medium for jth band component may be expressed as:

$$\frac{dI_j^m}{ds}=\kappa_j(a_j n^2 I_b - I_j^m) \quad (2)$$

where, I_j is intensity of radiation of jth band, I_b is intensity of the black body, m is discrete directions, s is path, κ is absorption coefficient, a is weighting coefficient and n is real part of the refractive index.

In the DOM model [6], the RTE (Eqn. 2) is solved for each band of glass emission. The total radiation intensity is calculated from

$$I=\sum_{j=1}^{N_b} I_j^m \quad (3)$$

The wall heat q_w on the surface for each grid point is calculated from the sum of all incoming intensities of all number of bands N_b, and direction m with discrete direction vector $\vec{\Omega}$ and normal vector $\hat{n}$

$$q_w=\sum_{j=1}^{N_b}\sum_{m=1}^{M} I_j^m(\vec{\Omega}.\hat{n})\,d\vec{\Omega} \quad (4)$$

where M is total number of rays.

The divergence of heat, which couples RTE to general energy equation, is

$$\nabla.q = \sum_{j=1}^{N_b} \kappa_j (4\pi a_j n^2 I_b - \sum_{m=1}^{M} I_j^m d\Omega) \tag{5}$$

In the KEFF approximation [1], the heat flux q is expressed by treating the heat transmission as a diffusion process:

$$q_{con+rad} = -k_{eff} \nabla.T \tag{6}$$

where k_{eff} is the effective thermal conductivity which is the sum of thermal and radiative conductivities. The heat losses from the walls to the ambient air may be expressed as

$$-k_{eff} \frac{\partial T}{\partial n} = U(T - T_{amb}) \tag{7}$$

where T_{amb} is ambient air temperature.

2.2 **Radiative properties of Glass Medium**

Figure 1 shows typical spectral variation of absorption coefficient of the glass medium over the bands as a function of temperature. Spectral data is obtained experimentally for differentiate glasses ranging from colourless to dark [5]. Variation of spectral absorption coefficient of glass bath can be divided into three bands where the absorption coefficient is allowed to change only with temperature.

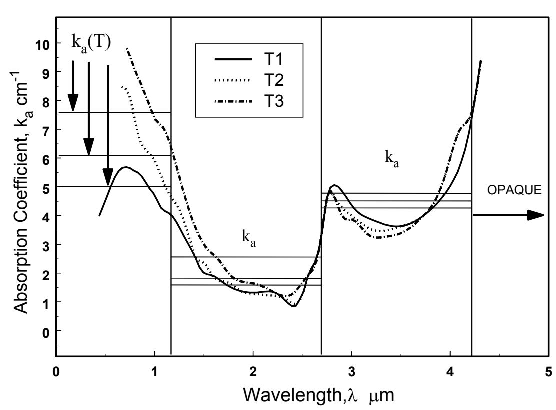


Fig 1. Spectral variation of absorption coefficient over the bands as a function of temperature (T1>T2>T3) [1]

3. APPLICATIONS
4.

The performance of DOM is tested for the following cases:

a) radiative equilibrium between isothermal plates in a 1-D gray planar medium,
b) conduction and radiation heat transfer in a 1-D non-gray planar glass medium and
c) conduction and radiation heat transfer in a 1-D non-gray planar glass medium having uniform source term.

The last case represents the use of electrodes in glass melt tanks. The temperature distribution around the electrodes needs to be prediction accurately.

3.1 Radiative Equilibrium between isothermal plates in a 1-D gray planar medium

The performance of DOM for gray planar medium is tested before the evaluation of the method for non-gray media. The homogenous radiating–conducting medium is absorbing and emitting. Benchmark results for this problem are available in [7]. S_8 (= 16 rays) and central-differencing scheme are used. It is seen from Fig. 2 that as optical thickness τ increases optically thick limit is reached. Temperature profile becomes linear.

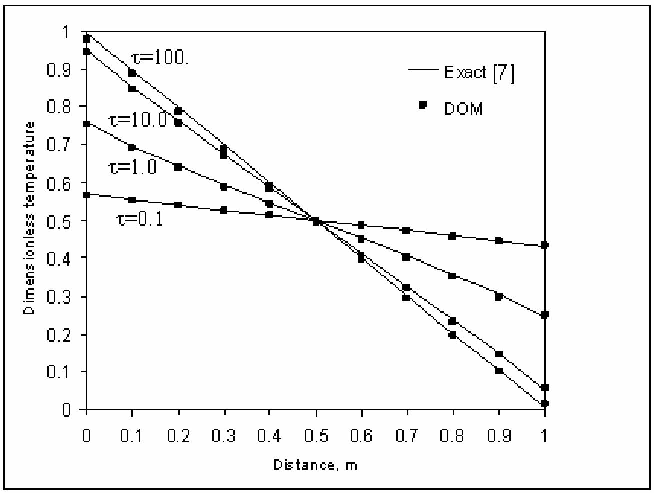


Fig. 2. Dimensionless temperature profile at different optical thicknesses τ .

Table 1 and 2 show dimensionless heat flux and temperature distribution at τ =100, respectively. As optical thickness increases percentage error in the prediction of heat flux distribution increases. Performance of DOM is also poor for the prediction of temperature distribution.

Table 1 Dimensionless heat flux distribution at different optical thicknesses τ.

Optical Thickness, τ	Exact [7]	DOM	% Error
0.1	0.9157	0.9134	0.25
0.4	0.7458	0.7439	0.25
1	0.5532	0.5532	0.00
4	0.2460	0.2459	0.03
10	0.1167	0.1169	-0.12
20	0.0622	0.0627	-0.67
50	0.0259	0.0274	-5.75
100	0.0131	0.0147	-11.91

Table 2. Dimensionless temperature distribution at τ =100

Distance, m	Exact [7]	DOM	% Error
0	0.9948	0.9776	1.725
0.1	0.8974	0.8863	1.234
0.2	0.7994	0.7873	1.509
0.3	0.6998	0.6885	1.611
0.4	0.6000	0.5899	1.682
0.5	0.5000	0.4915	1.705
0.6	0.4000	0.3932	1.694
0.7	0.3000	0.2952	1.613
0.8	0.2000	0.1973	1.361
0.9	0.1026	0.0996	2.948
1	0.0052	7.00E-03	34.61

3.2 Conduction and radiation heat transfer in a 1-D non-gray planar glass medium

The homogenous non-gray radiating–conducting medium is absorbing and emitting. Temperature distribution for combined radiation and conduction across a non-gray slab bounded by black plates with T_2= 1600K and T_1= 1100 K are calculated from band wise solution of RTE using the DOM method. Wavelength nature of refractory is not considered. Radiative properties of colorless and dark glasses are given below.

	Band limit, µm	Absorption coefficient of glass, κ, 1/m			
		Lighter	Light	Dark	Darker
Band # 1	1.20	0.3	3.0	30.0	300.0
Band # 2	2.70	0.3	3.0	30.0	300.0
Band # 3	4.20	0.3	3.0	30.0	300.0

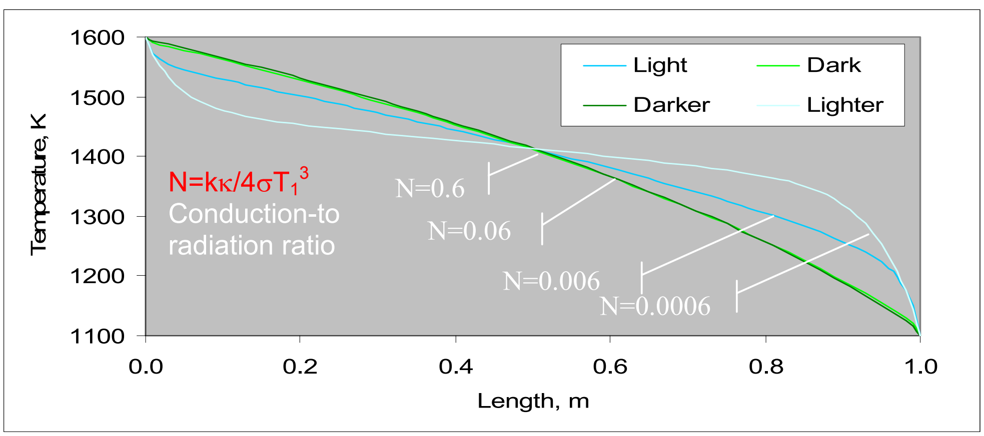


Fig. 3. Temperature distribution in a planar glass medium for four different glasses having as different conduction-to radiation ratio N.

Fig. 3 shows that as optical thickness of the medium increases the radiation becomes conductive process. Radiation model to be used for glass media should handle both optically thin and thick media.

3.3 Conduction and radiation heat transfer in a 1-D non-gray planar glass medium having uniform source term

Figure 4 describes the case for combined radiation and conduction across a non-gray glass medium bounded by isothermal black plates with Tw= 1500K and exposed to air at T_{air}= 300K and having overall heat transfer coefficient U = 2 W/mK. Uniform source term is placed in the middle of the medium. Qgen = 4.8 kW. This test case represents the use of electrodes in glass melt tanks. Radiative properties of colorless and dark glasses are given below.

	Band limit, µm	Absorption coefficient of green glass, κ, 1/m			
		Colourless	Dark		
Band # 1	1.20	20.0	1031.4	-1.2449 xT	6.5E-04 xT^2
Band # 2	2.70	20.0	729.7	-0.8711 xT	3.5E-04 xT^2
Band # 3	4.20	220.0	-222.0	0.8988 xT	-3.0E-04 xT^2

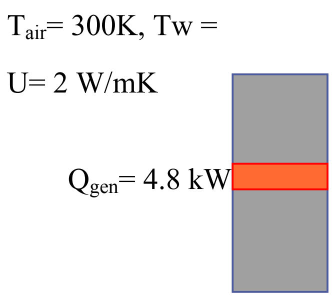
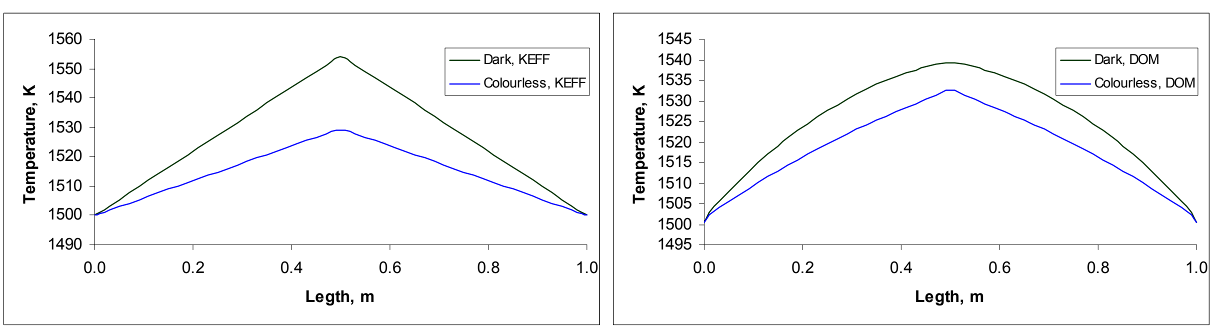


Fig. 4. Planar semitransparent medium having uniform source term.



(a) (b)

Fig. 5. Temperature distribution across the slab, calculated by (a) KEFF and (b) DOM for colorless and dark glasses.

The predictions of DOM are compared with those obtained by effective thermal conductivity approximation. Fig. 5 shows that DOM over predicts temperature distribution across the slab.

4. **CONCLUSION**

One-dimensional analysis of combined conduction and radiation is carried out in a planar semitransparent medium by using the DOM. Glass is taken as an example of a semitransparent medium. The predictions are compared with effective thermal conductivity approach, which is commonly used for the simulation of the glass melting tanks. The 1-D test case is chosen in a way that it represents the use of electrodes in glass melt tanks. It is found that although very high angular quadrature scheme, S_8 approximation, is used the DOM over predicts temperature distribution for dark glasses having high optical thickness. It needs to be modified for highly optically thick semitransparent media. It is planned as a future extension of the study.

5. REFERENCES

1. N. Kayakol, L. Önsel, Z. Eltutar and A. Ungan, Coupling Radiative and Thermal Fields in Cold Top furnaces, 18th ICG, San Francisco, USA (1998).

2. N. Kayakol, Discrete transfer and discrete ordinates methods for radiative heat transfer in industrial furnaces, Ph.D. Thesis, Middle East Technical University, Turkiye, 1998

3. M. Varady and A.G Fedorov, Combined radiation and conduction in glass Foams, Journal of Heat Transfer, Vol. 124, pp. 1103-1109 (2002)

4. K.Lee and R Viskanta, Prediction of spectral radiative transfer in a condensed cylindrical medium using discrete ordinates method, J. Quant. Spectrosc. Rodiat. Transfer Vol. 58, No. 3, pp. 329-345 (1997)

5. A. J. Fabers , Optical properties and redox state of silicate glass, C. R. Chimie Vol. 5 pp. 705–712 (2002)

6. N. Kayakol, An analysis of spectral radiative heat transfer in glass furnaces, Glass Days, 5-6, November, Eindhoven, Nederland, (2003)

7. M. A. Heaslet and R. F. Warming. Radiative transport and wall temperature slip in an absorbing planar medium. Int. J. Heat Mass Transfer, Vol. 8, pp. 979–994 (1964)