BOUNDARY ELEMENT METHOD EMPLOYING CARTESIAN HIERARCHICAL MESHES

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The paper is devoted to the application of the Boundary Element Method (BEM) to heat radiation problems. BEM can be seen as an alternative approach to the well established zoning method or FEM, its higher order generalization. When compared with these approaches, BEM offers substantial computing time economy due to the reduction of the integration dimension and lack of volume integrals[1]. Another distinguished feature of BEM is its sound mathematical background[2]. Earlier studies of Wecel and Bialecki[3,4] show that BEM is a robust tool of simulating radiative heat transfer. Authors have developed a method that can be applicable to wide range of industrial problems encompassing the entire range of optical densities. Good agreement with the results of other methods of solving RTE has been achieved with the execution times of the order of Discrete Ordinates[5].

The idea of BEM is first to integrate formally the radiative transfer equation (RTE) along the line of sight. The result is then integrated over the hemisphere centered at the point lying on the boundary of the enclosure. Integration over the solid angle is converted into integration over the surface of the radiating enclosure. The resulting integral equation links the radiative heat flux on the boundary with the blackbody emissive powers of the walls and the medium.

Heat transfer driving force is temperature difference. Since the radiative energy fluxes are proportional to the fourth power of the temperature even relatively low gradients of temperature lead to significant intensification of the radiative heat transfer. Hence problems with high temperature gradients require careful internal discretization in order to adjust the volumetric cell dimension to the gradients of the the temperature field.

Local mesh refinement can readily be introduced, in the framework of the general unstructured meshes. However, ray tracing on such meshes is too expensive. In the context of time economy, the *old-fashioned* Cartesian meshes are a much better alternative. It was shown[6] that, when compared with its unstructured counterpart, the ray tracing on Cartesian meshes leads to two orders of magnitude reduction of the execution time. Thus, the focus of this paper is on the improvement of the mesh quality within the framework of structural, Cartesian meshes.

It should be stressed that the volumetric meshes are used in BEM merely to ray tracing, not to evaluate volumetric integrals. Thus, these Cartesian grids need not be boundary fitted and they do not distort the behavior of the solution close to the boundary. The boundary itself is parametrized separately, completely independent from the volumetric mesh. Therefore, the discretization of the boundary can be carried out with practically arbitrary accuracy.

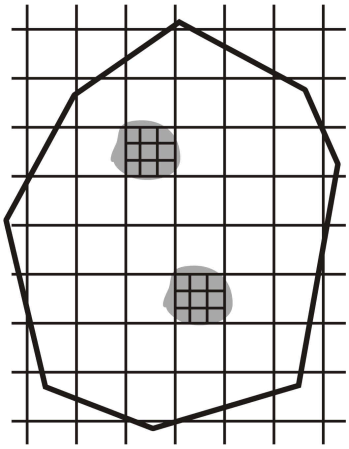
Figure 1 shows a possible 2D structure of the mesh. The high temperature gradient regions (hot spots) are marked as gray areas. In order to reproduce these regions with reasonable accuracy, the number of volume cells needs to be increased. The proposed technique of introducing a local refinement of the Cartesian mesh employs the idea of a hierarchic structure of the mesh. The higher level (coarse) mesh is generated first. Then a fine structured Cartesian mesh is introduced within selected coarse volumetric cells located in regions where rapid changes of temperatures are predicted. The ray tracing procedure on such a mesh is carried out hierarchically requiring some ingenuity in programming. In the first step, the ray is traced on the coarse mesh. The same ray tracing procedure is then repeated in cells where the ray passed finer mesh. Similar hierarchical structure has also the bookkeeping scheme used to store the results of ray tracing. It should be pointed out that as the volumetric mesh is used only for the ray tracing purposes, no special treatment of hanging nodes is required.

The developed procedure has been tested performing simulations of radiative heat transfer in real combustion chamber. In order to test procedure in high temperature gradient environment Oxy-Natural Gas flames[7] (pure oxygen as oxidizer) 3D calculation has been performed using commercial Fluent code. Oxy-Natural Gas flames is characterized by high temperature gradients in the flame front area. In the range of few centimeters temperature differences can reach even 1000 K. Hence fine enough grid has to be employed in the vicinity of this region. Such high gradient prevails form the tip of the burner till 1 m downstream along furnace axis and it has around 0.2 m in diameter. Remaining part of the chamber keeps relatively smooth temperature profile.

Number of different internal meshes have been generated in order to find proper refinement of flame area, when remaining part of the chamber covers rather coarse mesh. Firstly no adapted mesh has been employed in order to find maximum density required to properly resolve radiative heat sources. Not adapted mesh with division 19x19x27 in x,y, and z direction has been selected as the reference solution. X and Y direction has been condensed comparing to Z direction since gradient of temperature in these direction is higher. Non uniform distribution of volume mesh (more cells closer to the chamber axis and burner tip) helped better represent high temperature gradient area.

Next, couple of meshes with rather course division has been tested showing not enough mesh condensation of the flame area. Finally course meshes has been adapted with hierarchic meshes. Predicted radiative heat sources obtained with hierarchic BEM solver have been compared with results obtained by Discrete Ordinate Fluent code solver.

Figure 2 shows comparison of radiative heat sources obtained for 3 selected BEM meshes and DO solution. The results are printed on the profile aligned with Y coordinate direction, perpendicular to the chamber axis at the distance 0.5 m from the burner tip. Cases b1-b3 are BEM solutions with following settings: b1 - not adapted course volume mesh, around 2700 cell, b2 - not adapted dens mesh, around 10 000 cells, b3 - adapted course volume mesh (b1), around 4500 cells. DO simulations has been performed on CFD based mesh of around 1 milion volume cells. Cases b2 and b3 shows very good agreement with DO solution. Only mesh for case b1 has too coarse density in the flame area, and is not able to reproduce correctly radiative sources.



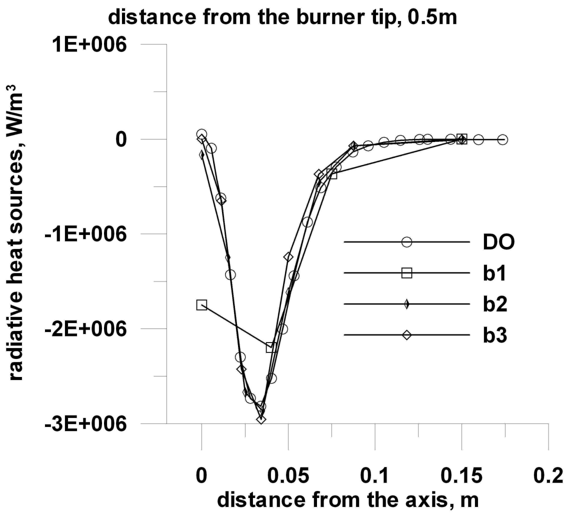


Figure 1. Cartesian hierarchical meshes in regions with hot spots.

Figure 2. The radiative heat sources.

The approach employs hierarchic structure of the two levels of the mesh in order to reproduce temperature field with higher accuracy. The method predicts radiative heat sources with acceptable accuracy at the reasonable time when compared to fine meshes simulations. Time reduction is significant, solution is about 5 times faster. The refinement technique presented here can be applied to any RTE solution method which utilizes Cartesian meshes and employs ray tracing method.

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